SHORTER NOTICES

The Expanding Universe. By Sir Arthur Eddington. New York, Macmillan, 1933. x+182 pp.

This volume, an enlarged version of a lecture delivered by Eddington at the meeting of the International Astronomical Union at Cambridge (Mass.) in Sept. 1932, treats the theory of the expanding universe, not as an end in itself, but as a means for determining the cosmical constant which appears in the Einstein theory. The subject lies at the meeting point of astronomy, relativity, and wave-mechanics. If the theory developed is valid, then (p. 170) "to measure the mass of an electron, a suitable procedure is to make astronomical observations of the distances and velocities of the spiral nebulae." This indicates the marked interest of the book for all those who delight to meditate on the interrelatedness of natural phenomena.

R. D. CARMICHAEL

Fourier'sche Reihen, mit Aufgaben. By J. Wolff. Groningen, P. Noordhoff, 1931. 60 pp.

This monograph, written in synoptic form, is an excellent condensation of material that is usually developed in many more pages. The choice of material is also commendable; the theorems selected are certainly among the most fundamental in the vast number of results concerning Fourier series available in classical and modern literature.

The book is divided into three sections, each accompanied by a valuable set of exercises. Many of the exercises, indeed, involve fundamental results in the theory of Fourier series which are sufficiently closely related to theorems in the text to justify having their development left to a student possessing initiative. The first two sections of the book can be read by those familiar only with the Riemann integral. The third section presupposes a knowledge of the theory of the Lebesgue integral.

Among results presented in the first section are criteria for convergence of the Fourier series for functions satisfying Lipschitz conditions, and functions which are monotonic, or of bounded variation, both in connection with convergence at a point and uniform convergence in an interval. There is also given an example of a continuous function whose Fourier series diverges at a point.

The second section contains a discussion of summability $(C1)$ of the Fourier series, the best approximation property, in the sense of least squares, of the Fourier partial sums, Fejér trigonometric polynomials, Parseval's theorem, some of Riemann's theory of trigonometric series in general, and some of Cantor's work on the uniqueness of Fourier developments. The Fourier integral is treated in the exercises.

The third section begins with the extension of classical results to Lebesgue integrable functions. Then, after certain properties of such functions are discussed, some of the recent results depending essentially on Lebesgue integration are obtained. There is a discussion of the convergence criteria due to de la Vallée Poussin, Lebesgue, and W. H. Young, and some of Fatou's results in connection with the Poisson integral. The Riesz-Fischer theorem and other important results are given in the exercises.

The book as a whole should certainly serve as an excellent adjunct to a course in Fourier series, both for students of pure mathematics and of mathematical physics. It could also serve well as a guide to a student attempting to make progress in the theory by means of individual study. In this case it would be advisable to use it in connection with other texts where the treatment is less condensed. Finally, it is an excellent reference work for those already familiar with the theory who desire a compact summary of the most outstanding results.

C. N. MOORE

Einführung in die Theorie der kontinuierlichen Gruppen. By Dr. Gerhard Kowalewski. (Mathematik und ihre Anwendungen, Band 9.) Leipzig, Akademische Verlagsgesellschaft, 1931. 10+396 pp.

This attractive volume is a distinctly worth while addition to the literature of the Lie theory of continuous groups. The author, himself an erstwhile student of Lie's, has lectured at various times on this subject at a number of German universities, and from these courses, and from a basis of notes and recollections of conversations with Lie, the present work is professed to have sprung. The presentation is made with care and skill and is distinctly readable. With the introduction of each new concept the author avails himself of illustrative material from geometry or mechanics to help set forth the idea itself preparatory to its formulation in analytical garb. Thus, at the very beginning a consideration of the steady two-dimensional flow of a fluid is made to yield the concepts of the group of infinitesimal transformations, the path curves, the integrated finite transformations, etc. Critical remarks are freely interspersed in the text and add life to the presentation. At appropriate intervals the theory is summarized, and is consolidated now and again by detailed and sometimes lengthy discussions of important and interesting special cases.

Despite the smoothness of its exposition the book requires of the reader a considerable measure of mathematical maturity. A familiarity with the analysis necessary and incidental to the discussion is assumed without question. The work is not to be thought of, therefore, as one designed or suited to give a hasty and superficial acquaintance with the subject. On the contrary it seeks to present the theory in as complete a form as possible, and to this end the author has sought to incorporate with the fundamental work of Lie the important results of such modern investigators as Cartan and F. Schur.

The volume is one of 394 pages and is divided into four chapters. The first of these, entitled *Infinitesimal Transformations and One-Parameter Groups,* begins with the fundamental definitions and extends through a discussion of the integration of Lagrangian and Pfaffian systems of equations. The remaining chapters are given respectively to *Multiple-Parameter Groups and Their Infinitesimal Transformations,* to *Lie's Fundamental Theorems,* and to *Groups of Transformations on the Line and in the Plane.*

The book will be indispensable to workers in its field, and is generally to be recommended.

R. E. LANGER