NOTE ON A PREVIOUS PAPER

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Dr. Hillel Poritsky has called my attention to the fact that the second theorem in my previous paper* can be treated more completely by the method of integral equations. No restriction concerning the existence of a first derivative of the force function $f(x)$ need be made, under the assumption of a constant period. We wrote

$$T(a) = 2\cdot 2^{1/2}\int_0^a \frac{dx}{(F(a) - F(x))^{1/2}},$$

where T was the period and a the amplitude, with $F(a) = \int_0^a f(x)dx$. Under the transformation $z = F(x)$, $h = F(a)$, $u(z) = 1/f(x)$, this becomes

$$T(a) = 2\cdot 2^{1/2}\int_0^h \frac{u(z)dz}{(h - z)^{1/2}} = 2\cdot 2^{1/2}\phi(h).$$

Under the hypothesis of constant period (and $f(x)$ non-vanishing for x greater than zero) we have $\phi'(h) \equiv 0$, and $\phi(h) \equiv \phi(0)$. The solution of the integral equation

$$\phi(h) = \int_0^h \frac{u(z)dz}{(h - z)^{1/2}}$$

gives

$$u(h) = \frac{1}{\pi}\cdot\left[\frac{\phi(0)}{h^{1/2}} + \int_0^h \frac{\phi'(z)dz}{(h-z)^{1/2}}\right],$$

which leads to

$$f(x) \equiv (4\pi^2/T^2)x.$$

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* *Three theorems applicable to vibration theory*, this Bulletin, vol. 38 (1932), pp. 718–723.