OSGOOD ON FUNCTION THEORY, II, 2

Lehrbuch der Funktionentheorie. By W. F. Osgood. Zweiter Band, zweite Lieferung. Leipzig, Teubner, 1932. 9+377 pp.

This second section of Volume 2 of Osgood's well known treatise on function theory is devoted to Abelian integrals and periodic functions. The pagination and chapter numbering are continued from the first "Lieferung." The material here treated falls into five chapters, namely: IV. Algebraic functions and algebraic integrals. V. The existence theorem. The integrals and the prime functions. VI. Abel's theorem. Intersection theorems. Theorem of Riemann-Roch. VII. Periodic functions. VIII. Applications. These titles suggest their own story. There is, however, no steady progress from one chapter to the next since one may think of the behavior of the normal integrals upon the dissected Riemann surface as being the essential topic in each of the chapters. (Tables of periods are displayed no less than 45 times.) In Chapter IV (the first of this "Lieferung") the algebraic form $G(w, z)=0$ is treated both as a plane curve and as a Riemann surface. The discussion is confined as is fitting to cases in which multiple points are of the simplest type. The integrals are set up, their properties studied, the periods are normalized, the bilinear period relations of Weierstrass and of Riemann are established and compared. The treatment in this chapter is the traditional one save for more than usual (but much needed) discussion of the number of intersections of algebraic curves in the projective plane and an exposition of the theory of the plane of function theory. In Chapter V, the methods of logarithmic potentials are applied. Prime functions are introduced and the normal integrals are thereby expressed by use of differentiation. The conformal mapping established by inversion provides for an automorphic group whose invariants interpret the functions upon the Riemann surface. One returns to geometry in the study of Noether's normal curve and of its projections. The abstract treatment in this chapter and the extensive list of necessary special notations would make the reading at best difficult to most students. The avoidance of illustrative material tends to render the topic unnecessarily forbidding even although the reader will appreciate the higher view points provided for topics mentioned in the previous chapter. Chapter VI is devoted to Abel's theorem and the theorem of Riemann-Roch, both suggested earlier. The essentially new topics concern the use of homogeneous differential expressions. The adjoints of order $m-3$ are again discussed. The prime function is studied in the homogeneous form. There is additional material on space curves. As with other chapters illustrative material is scanty unless one is to view the entire chapter as essentially a review of the two preceding in terms of homogeneous variables. Chapter VII reverses the historical sequence and returns to the point of view of Jacobi. It approaches the problem of inversion, by the study of periodic functions in p arguments. A third of the chapter is covered by the normalization of the periods and related topics. Then the theta-functions are introduced as infinite series. A brief mention is made of characteristics, and the problem of normalizing the periods of the theta-functions is treated. Chapter VIII takes up the correspondence principle for points on an algebraic curve, and the 28 double tangents of a quartic, and closes with five pages on the Riemann-Weierstrass theta-theorem.

There is little new in this book beyond the occasional use of the plane of function theory, nor is it encyclopedic. Although much of the language suggests geometry, mention is made only of those geometrical concepts which are strictly essential to the topic. The domain of variation is sometimes viewed as a curvilinear polygon, sometimes as a dissected Riemann surface, sometimes as a plane curve, sometimes as a curve in n-space. It is never the mere abstract bearer of algebraic groups of points, as handled successfully by the modern Italians. The pervading points of view are those of Klein with wordiness excised and with more explicit proofs. Charm and thrill are sometimes found in proceeding from the simple to the advanced, from the concrete to the abstract, in the developing conviction of power to handle problems, in catching glimpses of domains yet awaiting the adventurer. These qualities often apparent in the original papers of the investigators, and retained in such expositions as that of Appell and Goursat, are here consciously sacrificed to an impersonal economy in abstract and logical treatment which may be justified if this interesting subject is to be compressed to fit into a general treatise on function theory.

A. A. Bennett

VEBLEN-WHITEHEAD—FOUNDATIONS OF DIFFERENTIAL GEOMETRY

The Foundations of Differential Geometry. By Oswald Veblen and J. H. C. Whitehead. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 29.) Cambridge, at The University Press, 1932. New York, The Macmillan Company. ix+96 pp.

In the preface to his earlier work on *Invariants of Quadratic Differential Forms* (Cambridge, 1927), Professor Veblen regretfully stated that even a short discussion of differential geometry had been crowded out because of the limited space available. If the necessity for this omission led to the writing of the present tract, it was a fortunate circumstance, for the result is a profoundly stimulating book.

The title, *The Foundations of Differential Geometry*, is scarcely broad enough to describe the contents, for the book is a critical study of the foundations of all mathematical systems which have been called geometries. The authors have kept strictly to their theme, namely the logical foundations of geometry, and have not yielded to the temptation to develop any particular geometry much beyond the postulates by which it is defined. While the treatment is philosophical and highly critical, yet it is so skillfully done that the book is easy to read—a fact which testifies to the genius of the authors for mathematics of this kind.

A rigid definition of geometry is not attempted, on the ground that any objective definition of geometry should include the whole of mathematics. The question is dismissed with the statement that a branch of mathematics is called a geometry because the name seems good, on emotional and traditional