One is thus led to the conclusion that

$$\lim_{a\to 0} 2\cdot 2^{1/2}\int_0^a (F(a) - F(x))^{-1/2}dx = 2\pi b^{-1/2}.$$

Hence we have the following theorem.

THEOREM 3. *The period of vibration T under restoring force $f(x)$, conditioned by hypotheses* (A), (B), *and* (C), *approaches the limit $2\pi b^{-1/2}$ as the amplitude approaches zero.*

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A NOTE ON FERMAT'S LAST THEOREM

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In 1925 H. S. Vandiver† proved the following theorem.

THEOREM 1. *If*

$$x^p + y^p + z^p = 0 \tag{1}$$

is satisfied by integers x, y, z, prime to the odd prime p, then the first factor of the class number of the field generated by $e^{2\pi i/p}$ is divisible by p^8.

In the seventh of a series of articles on Fermat's last theorem, T. Morishima‡ has given the following improvement upon Theorem 1.

THEOREM 2. *In Theorem* 1 *we may replace p^8 by p^{12} provided p does not divide* $75571\cdot 20579903$.

It is the purpose of this note to show that the proviso of Theorem 2 is unnecessary by showing that (1) is not satisfied by the prime factors of $75571\cdot 20579903$. This is done by applying Wieferich's|| criterion.

THEOREM 3. *If* (1) *is satisfied by integers x, y, z, prime to p, then* $2^{p-1}\equiv 1 \pmod{p^2}$.

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† Annals of Mathematics, (2), vol. 26, p. 232.

‡ Proceedings of the Imperial Academy of Japan, vol. 8 (1932), pp. 63–66.

|| Journal für Mathematik, vol. 136 (1909), p. 203.

In the first place

$$p_1 = 75571 \text{ and } p_2 = 20579903$$

are prime numbers. To show that p_2 is a prime we observe that $2^{(p_2-1)/563} = 2^{36554} \equiv 18351241 \equiv r \pmod{p_2}$ and that $r-1$ and p_2 are relatively prime. By the congruence (2) below we have

$$2^{p_2-1} \equiv 1 \pmod{p_2}.$$

Hence all factors of p are of the form $563x+1$.* There are no primes of this form less than the square root of p_2. Hence p_2 is a prime.

We find next that

$$2^{p_1-1} \equiv 4481813727 = 1 + 59306p_1 \pmod{p_1^2}$$

and

$$(2) \quad 2^{p_2-1} \equiv 70637882819917 = 1 + 3432372p_2 \pmod{p_2^2}.$$

Hence, by Theorem 3, equation (1) has no solutions x, y, z, prime to p for $p=p_1$ or p_2. We have then the following lemma.

THEOREM 4. *If $x^p+y^p+z^p=0$ has a solution for which xyz and p are coprime, then the first factor of the class number of the cyclotomic field $K(e^{2\pi i/p})$ is divisible by p^{12}.*

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* See this Bulletin, vol. 33 (1927), p. 331. Theorem 3.