Outline of the History of Mathematics. By Raymond Clare Archibald. Brooklyn, The Society for the Promotion of Engineering Education, 1932. 53 pp.

This *multum in parvo* of mathematical history forms No. 18 in the Selected Papers of the Summer School for Engineering Teachers issued by the Society for the Promotion of Engineering Education. It is reprinted from the Journal of Engineering Education (New Series, vol. 22, pp. 246–298, Dec., 1931) and consists of two lectures delivered before the Mathematical Session of the Summer School for Engineering Teachers at the University of Minnesota, August 31 and September 1, 1931.

The first lecture, History of Mathematics before the Seventeenth Century, covers a period of about 5000 years and the second, History of Mathematics after 1600, a period of about 300 years. The author accurately characterizes the work (p. 1) as an attempt "to give indications of the development of mathematics before the nineteenth century, and to refer briefly to some developments of the nineteenth century in connection with topics usually discussed at undergraduate colleges. Chinese and Japanese mathematics are not considered in such a skeleton survey, and the reference to mathematics of the Hindus is brief."

Since summaries and outlines are, of necessity, quite general in their nature and usually prepared to meet the needs of beginners in the field, it might be supposed that good ones are easily written. The opposite is the case. The preparation of a scholarly outline requires a large range of information, good judgment in selection of material, patience in checking carefully many details, and familiarity with the results of recent research. Fortunately, the author of this outline measures up to these qualifications. The reader may rest assured that he will find material well selected and carefully documented and that he will not find repetitions of errors which have, all too often, persisted in general secondary works long after research workers had published corrections.

U. G. MITCHELL

Vorlesungen über einige Klassen nichtlinearer Integralgleichungen und Integrodifferentialgleichungen nebst Anwendungen. By Leon Lichtenstein. Berlin, Julius Springer, 1931. x+164 pp.

This monograph contains extremely rich material and introduces the reader into many theories of high importance. It represents a modified and magnified reproduction of a course of lectures delivered in 1931 by invitation at the University of Lwów. The first chapter (Nichtlineare Integralgleichungen im kleinen) contains an existence and uniqueness proof for a general class of non-linear integral equations, which includes that treated in well known papers by E. Schmidt as a special case. Instead of Schmidt's power series expansions the author consistently and successfully uses an elaborated method of successive approximations. Special attention is given to the investigation of the case of ramification (Verzweigungsfall) which plays a fundamental role in the theory and in its applications. Various extensions (for example, to integro-differential equations, systems of integral and integro-differential equations, etc.) are briefly treated. Chapter 2 gives immediate applications of the preceding theory to various problems of mathematical physics and partial differential equations. We mention particularly the elegant solution of the problem of propagation of two-dimensional surface waves of a finite amplitude, treated by entirely different methods by Levi-Civita and several other writers; also a non-linear problem of heat conduction generalizing the one treated previously by Carleman. The end of the chapter is devoted to a discussion of solutions of the elliptic partial differential equation $\Delta z = F(x, y, z, \partial z/\partial x, \partial z/\partial y)$ considered as functionals of their boundary values. Integro-differential equations of a more general type, that can not be reduced to integral equations, are discussed in Chapter 3. Here we find a discussion of a general elliptic partial differential equation of second order containing a parameter, with applications to regular problems of the calculus of variations. Numerous applications of a general inversion principle in the theory of functional equations of a certain type are given to the problems of figures of equilibrium of rotating fluids, to dynamics of completely incoherent gravitating media, and to hydrodynamics of homogeneous incompressible perfect fluids. Chaper 4, the last, contains an application of the method of W. Ritz to the proof of the existence of characteristic values of an integral equation of the form

$$\lambda\phi(s) = \sum_{n=1}^{\infty} \int \cdots \int K_n(s; s_1, s_2, \cdots, s_n)\phi(s_1) \cdots \phi(s_n) ds_1 \cdots ds_n + f(s),$$

and of some of its modifications.

Various bibliographical references given in the book will be found very valuable by the reader. It is gratifying to find emphasis on the importance for the theory in question of numerous investigations by Liapunoff, who should be considered as a real founder of the theory of non-linear integral equations, a fact which has been partially or completely overlooked by many a writer of high authority. It is surprising, however, that important investigations of N. Günther in the existence problem of equations of hydrodynamics of a perfect fluid are not mentioned, and even that any reference to Günther is absent in the monograph.

The appearance of the book is excellent, although, even at the first reading, the reviewer was able to find quite a few misprints and lapses. On one point of some importance the reviewer finds himself to be in disagreement with the author. We have no doubt that for an intelligent reading of the book, particularly of Chapters 2 and 3, the reader will need a thorough knowledge of potential theory and of the theory of linear elliptic partial differential equations, including even quite recent developments, and many other things in addition, contrary to the author's statement in the Introduction that the "Elements of potential theory and of linear elliptic partial differential equations are desirable but not necessary, since everything that is needed is thoroughly explained in detail."

J. D. Tamarkin