CARTAN ON COMPLEX PROJECTIVE GEOMETRY

Leçons de Géométrie Projective Complexe, par E. Cartan. Paris, Gauthier-Villars, 1931. vii+325 pp.

This volume has been prepared from the notes of M. F. Marty on a course of lectures given by Cartan at the Sorbonne during the winter of 1929–1930. As Cartan is one of the foremost geometricians of our time, one may, of course, expect that exposition and content of the treatise bear the marks of a master in his field. This is indeed the case, so that one may state without exaggeration that the book under review contains the most comprehensive and scientific treatment of complex geometry in existence.

It is hardly necessary to emphasize the importance of geometries in a complex domain. This is evident when we consider for example the transformation

$$z' = \frac{az + b}{cz + d}\cdot$$

When a, b, c, d are real constants and z', z real variables, we are concerned simply with real projective geometry on a line, with its relatively simple content. But let the constants and variables involved be chosen from the complex domain, so that we are now concerned with the projective geometry of the complex line. Everybody realises how enormously the field has been enlarged and enriched. We need to think of the interpretation by circular transformations in the complex plane only, and its important applications in various mathematical fields, to realize the utility of extending geometric investigation into the complex domain.

The same relative importance attaches to the extension to higher complex spaces as carried out in a masterly fashion by Cartan. Assuming all quantities chosen from the complex domain, one may say that the content of projective geometry in the complex domain is formed by those properties of geometric forms or varieties which are invariant under the projective transformation or collineation (*homographie*)

$$\begin{aligned} x_1' &= a_{11}x_1 + \cdots + a_{1n+1}x_{n+1}, \\ x_2' &= a_{21}x_1 + \cdots + a_{2n+1}x_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+11}x_1 + \cdots + a_{n+1n+1}x_{n+1}, \end{aligned}$$

or the projective anti-collineation (*antihomographie*)

$$\begin{aligned} x_1' &= a_{11}\bar{x}_1 + \cdots + a_{1n+1}\bar{x}_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+1\,1}\bar{x}_1 + \cdots + a_{n+1n+1}\bar{x}_{n+1}, \end{aligned}$$

in which $\bar{x}_i$ signifies the conjugate of x_i, and $|a_{ik}| \neq 0$.

The latter were introduced and studied by Juel and chiefly by Segre in 1889.

In his lectures Cartan presents the fundamental notions of these complex geometries and gives interpretations which connect them with Riemannian geometry. The work is divided into two parts. The first is given to the projective geometry of the complex line and its relation to hyperbolic (Lobatchewsky) geometry. The second is concerned with complex geometry in three dimensions. The last chapter deals with harmonic polynomials of complex projective space and their application to the representation of this space or of elliptic Hermitian space, by real algebraic varieties without singularities imbedded in a euclidean space of a convenient number of dimensions.

These few indications on the contents, which are altogether too extensive to give in detail, should convey an idea of the nature of Cartan's *Leçons*. Their reading does not require more than ordinary mathematical preparation, a knowledge of the cross ratio and a notion of Riemannian space.

To anyone who is interested in this field with a view to research, Cartan's book may be highly recommended as an indispensable background for further progress.

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NEW MATHEMATICAL TABLES

Mathematical Tables. Vol. 1. Prepared by the Committee for the Calculation of Mathematical Tables. London, British Association for the Advancement of Science, 1931. xxxv+72 pp.

Standard Four-Figure Mathematical Tables. By L. M. Milne-Thompson and L. J. Comrie. London, Macmillan and Company, 1931. xvi+245 pp.

The first activity of the British Association for the Advancement of Science in the preparation of mathematical tables appeared in a report of a committee published in 1873. "The purposes for which the Committee was appointed are twofold, viz. (1) to form as complete a catalogue as possible of existing mathematical tables, and (2) to reprint or calculate tables which are necessary for the progress of the mathematical sciences." This Committee, with changes in personnel, has been active for most of the time during the last sixty years. In the lists of members are to be found the names of Cayley, Stokes, Sir W. Thomson, Glaisher, Lord Rayleigh, Greenhill, Sylvester, Pearson, and many other well known British mathematicians. The only American to take part in the work as a committee member appears to have been A. G. Webster.

It was at first intended that tables should be published independent of the annual Report of the Association, and some tables have been so published, but most of the results have been included in the annual reports. The Association is now carrying out the original plan of separate publication, the first step of which is explained in the following quotation from the preface. "For several years the question of collecting into book form the tables from its reports has been before the Committee, but it was apparent from the first that the simple plan of reprinting existing material would produce a heterogeneous volume neither useful nor creditable. There were gaps in the ranges of the arguments of some of the functions, natural when the tabulation had been performed at