Geometry of Four Dimensions. By A. R. Forsyth. Cambridge University Press, 1930. Two Volumes, Vol. I, 468 pp., Vol. II, 520 pp.

In the past few years several books on the geometry of four dimensions have appeared. They have been written, usually, with the aim of presenting the geometry necessary for the understanding of the theory of relativity. The present book has no such end in view. The author wishes simply to present a complete account of that geometry. The question then arises, why four dimensions and not n dimensions. We are told, in the preface, that there are two reasons for this; first, four dimensions is the simplest generalization of three dimensions and by treating these generalizations in minute detail much may be gained; second, there is a traditional importance to four dimensions.

One might naturally suppose that these two volumes would contain all that is known about the geometry of four dimensions, but this is not the case, as one more or less familiar with the subject soon finds out. The general plan of the work is excellent and it is written in the characteristic Forsyth style. The method used is that of Gauss and the notation is that of Cayley. The treatment is wholly analytical. The space is euclidean and in the preface we are told that curvature of a curve measures its departure from a euclidean straight line and that the complete curvature of a surface is a measure of its deviation from an euclidean plane. It seems to me that these statements need considerable interpretation since a fixed plane, for example, is euclidean or not according to the measure used. The only reason I can see for assuming euclidean space to be at the bottom of things is because it is the simplest known.

The work is divided into five main parts:

I. The first part (203 pages) is an account of the geometry of lines, planes and hyperplanes and the relations of one to another. I know of nowhere else that this material can be found in one place. A complete discussion of parallelism and perpendicularity of these elements and of the shortest distance and angles between them is given. A chapter on rotations in four dimensions is also included.

II. Part two (147 pages) is devoted to the theory of curves. The treatment is quite analogous to the theory of curves in ordinary 3-space but we have a new curvature, the second torsion or tilt, which complicates the theory to some extent. A chapter on curves in n-space is also included.

III. Part three (115 pages) is on surfaces in 4-space. While some parts of the theory of surfaces in 4-space are quite analogous to that of surfaces in 3-space there are other parts of it which are quite different. Geodesics are analogous, but properties of ordinary surfaces which depend upon the normal do not generalize quite so easily. Lines of curvature and asymptotic lines are examples. Gaussian curvature is the same; but in four dimensions, mean curvature is a vector. There are other properties which depend upon the fact that four is an even number and that the dimension of the surface is half this number. These topics are all adequately treated in this part of the book.

IV. Part four (381 pages) is on hypersurfaces. Geodesic lines, lines of curvature, and asymptotic lines are easy generalizations from ordinary surface theory. Curvature however is again quite different. The product of the principal radii is quite different from the product of the principal radii of curvature of a surface in 3-space. The Riemannian curvature of a hypersurface in 4-space has particularly interesting properties due to the fact that any orientation can be defined in terms of three, which is equal to the number of dimensions of the hypersurface. Many special hypersurfaces are also of considerable interest; among these may be mentioned hyperspheres, ellipsoids, and minimal hypersurfaces. This is only a small indication of the many things treated in this part of the book.

V. Part five, Invariants (120 pages). A book on this subject that makes any claim to completeness must necessarily discuss this subject. The treatment is based on Lie's continuous groups and covers the subject quite adequately.

C. L. E. MOORE

The Logic of Discovery. By R. D. Carmichael. Chicago, The Open Court Publishing Co., 1930. 274 pp.

Thanks to the analyses of both mathematicians and philosophers the science of logic has enjoyed greater development in the last seventy-five years than in the entire twenty odd centuries of its previous history. Up to 1850 it was identified with the formal logic of Aristotle. Since then five generalizations beyond Aristotle have occurred, each one of which has produced a new and more fertile system of logic. These systems are (1) the logic of classes initiated by Boole and Schröder, (2) the logic of relations associated with the names Peirce, Royce, Russell and others, (3) the logic of propositional functions formulated by Russell, (4) the logic of systems or doctrines and system or doctrinal functions, recently developed into an even higher generalization by Sheffer, and (5) the calculus of propositions first formulated by Russell and Whitehead in the *Principia Mathematica*, recently reinterpreted by Wittgenstein, and modified and perhaps even transcended by Hilbert, C. I. Lewis, Weyl, and others.

Carmichael's *Logic of Discovery* is based upon the fourth of these generalizations, the logic of systems or doctrines. Nothing new concerning this logic appears in his book. Its originality consists in its specification of the changes in our conception of mathematics, of reasoning, and of scientific method in the physical and social sciences, which the established concepts of this logic entail.

The well known fact that logical inference in a doctrinal function depends solely on the form is brought into the foreground. The key to this is really the notion of the variable and the discovery of the formal properties of relations which came into the logic of systems from the logic of propositional functions and relations. Carmichael's use of an experiment carried on by Veblen is most convincing on this point. The equivalence of postulate sets for the same system or system function is also indicated. These considerations enable Carmichael to reveal the relativity of the ordinary linguistic statements of a system, and the formal and purely rational side of scientific procedure.

His most original contribution concerns our conception of scientific method. It has been known for a long time by all acquainted with the historical evidence that Bacon's naive theory of induction has very little to do with actual scientific procedure, particularly as it occurs in the most mature sciences. Recently it has become the custom to conceive of all scientific work in terms of the method of hypothesis. But both conceptions have a common presupposition: Scientific statements are considered in terms of a few isolated propositions. The logic of systems and the part which it plays in mathematical physics forces a modifica-