EDGE ON THEORY OF RULED SURFACES

The Theory of Ruled Surfaces. By W. L. Edge. Cambridge, University Press, 1931. ix+324 pages.

The appearance of the present volume, written by a competent author and published by a recognized house, furnishes further evidence of the general re-awakening of interest in algebraic geometry. The purpose is to describe the algebraic ruled surfaces of orders three to six, by means of the configuration of the double curves and of the developables of bitangent planes.

Only a limited knowledge of projective and analytic geometry is presupposed; a general introduction to the theory of correspondences, geometry on an algebraic curve, and the general properties of ruled surfaces is provided in the Introduction. The plan there is to state the essential theorems, frequently with illustrations, then to give either the reference to its source or to its demonstration in a well known book. The proofs in this preparatory chapter are not included. Two methods of procedure are followed; either one would be sufficient to accomplish the purpose, but when one becomes difficult to apply, the other may suggest new phases of attack. This double derivation provides a comprehensive check on the accuracy of the result.

The first method consists in mapping the lines of ordinary space S_3 on the points of a general quadric manifold M_4^2 of four dimensions and order two in S_5, as proposed by Klein. Ruled surfaces of S_3 are then represented by curves of the same order and same genus on M. The latter are classified according to the dimensionality of the space in which the curve lies; then, if in S_4, according to the number of their intersections with the generating planes of the two systems on M; then, within each category, by the number of actual multiple points. Thus, if C lies in the section of M by a general S_4, the ruled surface belongs to a non-special linear complex, and is properly self dual, that is, each generator is transformed into itself under polarity as to this complex. If the S_4 is tangent to M, the polarity does not exist, and the ruled surface has a rectilinear directrix, which may or may not also be a generator, according as the curve does or does not pass through the point of contact.

No sufficient criteria have been developed to account for all particular forms of the curve which give rise to ruled surfaces having tacnodal or oscnodal multiple curves. With the exception of those surfaces contained in a linear congruence with coincident directrices, no attempt is made to include such cases; indeed their existence is not even mentioned.

The other method is the projection of a normal ruled surface from a higher space, as given by Segre. This is entirely practicable for rational ruled surfaces and for others of low orders, but becomes very cumbersome for higher genera. In the extreme cases the determination of the normal surface reduces to the very problem it is proposed to solve. The methods of projection into S_3 suggest various correspondences by which the surfaces may be generated. Frequently these are not the easiest nor the most rational methods of generation. A chapter on developables includes the proof that all developables of orders less than 8 are rational. The references to existing literature are much fuller in this chapter than in the others. The curves on M are images of developables when their tangent lines also lie on M.

A chapter on corrections and additions to the earlier types develops the suggestions for finding new ones that are brought out by the chapter on developables. Then follows a table of all the surfaces found, arranged as to order, then genus, then as to multiple directrices which may or may not be also generators, the main basis of classification, within each genus, being the configuration of double curves and of bitangent developables. Finally, a short discussion of interesections of curves on a ruled surface is added.

The book is admirably written, and faultlessly printed. The symbolism is consistent and very helpful. The only typographical error noticed is in the footnote of page 29. The year should be (1895) instead of (1865). The book is not provided with an index.

This excellent treatise contributes a real advance to the theory of ruled surfaces. Although both methods employed were known before, they have been here systematically developed and compared with each other to such an extent as to justify amply the publication of the book.

Two claims (see preface and page 139) made by the author need some explanation. The enumeration is said to be complete, and "so far as is known, no serious attempt has been made to solve this problem before." Since the references to various theorems are extensive, the following should be included: The first algebraic proof that a necessary and sufficient condition that a ruled surface be properly self dual is that it belong to a non special linear complex was given by C. H. Sisam (this Bulletin, vol. 10 (1903–04), pp. 440–441). The same author proved that every rational quintic ruled surface has three coplanar generators (this Bulletin, vol. 10 (1903–04), pp. 32–34). But the fundamental theorem concerning the number of component double curves, the genus of each and the number of its intersections with a generator furnishes a complete criterion for classification so far as rational surfaces is concerned (Sisam, American Journal of Mathematics, vol. 28 (1906), pp. 43–46). A number of omissions would have been noticed by use of this theorem. As to quartics, the comprehensive enumeration of Holgate (American Journal of Mathematics, vol. 15 (1893), pp. 344–386; vol. 22 (1900), pp. 27–30) should be cited. As to quintics, an enumeration including all those in the present volume is in this Bulletin (vol. 8 (1902), pp. 293–296). More details concerning those types having three double conics is found in this Bulletin (vol. 9 (1903), pp. 236–242). Two of the double conics may be replaced by a tacnodal one, or all three by an oscnodal (this Bulletin, vol. 11 (1905), pp. 182–186).

As to sextics, the list of K. Fink (Tübingen doctor dissertation, 1887) and that of J. Bergstedt (Lund dissertation, 1886) should have been mentioned, although neither would seriously affect Mr. Edge's statement. But the essay of A. Wiman, Klassifikation af regelytorna af sjette graden, Lund dissertation, 1892, contains 118 forms, including all mentioned in the present volume, and a considerable number of others. The basis of classification is different, but a wealth of detail concerning double curves and bitangent developables is given. It is synthetic, and maps the generators of the surface on the points of a curve, in ordinary space. In a series of papers in the American Journal of Mathematics, vol. 25 (1902), pp. 59–84, 85–96, 261–268; vol. 27 (1905), pp. 77–102, 173–188, the present reviewer established the existence and derived the equations of about two hundred sextic ruled surfaces, including all those in the book under review, and about eighty others which under Mr. Edge's basis of classification should be counted as distinct types. No criterion was found, to establish the completeness; thus far, no other forms have been published.

Of the specific omissions, the following have distinct double curves:

two double cubics and two double conics	$p=0$;
four double conics+two double generators	$p=0$;
double cubic and three double conics	$p=1$;
double quartic, two double conics and a double generator	$p=1$;
double quartic, double conic, double directrix and two double generators	$p=1$.

Then there are fourteen other forms without rectilinear directrices, having tacnodal or oscnodal curves. Thus, of the four double conics, two may approach coincidence, forming a tacnodal conic, or three may approach coincidence, forming an oscnodal conic. Of the large number of omissions of those having a directrix line which may or may not be a generator, a frequent sample is that caused by a compound involution.

For ruled surfaces having a directrix line, the reviewer is inclined to feel that the methods of Wiman and of Sisam, American Journal of Mathematics (vol. 29 (1907), pp. 48–100), are at least as powerful and as comprehensive as those developed by the author. Since all these papers are featured in the Encyklopädie (III C 8; Art. 52), the author should at least have mentioned them. There, if the reviewer has established his reasons for protesting the content of one paragraph, he now wishes to emphasize that Mr. Edge has produced an excellent book that will be of very great value in the study of various branches of algebraic geometry.

VIRGIL SNYDER

HITHERTO UNPUBLISHED TREATISE OF STEINER

Allgemeine Theorie über das Berühren und Schneiden der Kreise und der Kugeln worunter eine grosse Anzahl neuer Untersuchungen und Sätze vorkommen in einem systematischen Entwicklungsgange dargestellt. By Jakob Steiner. Edited by Rud. Fueter and F. Gonseth. Zürich and Leipzig, Orell Füssli, 1931. 8vo, xviii+345 pp. Price in marks: paper, 10.80; cloth 12.80.

This is an original manuscript written by Steiner more than a century ago. It now appears in print for the first time. It is published under the auspices of the Swiss Naturalist Society with the asistance of the Escher-Abegg Foundation for Scientific Research at the University of Zürich. It appears as volume five in a series of publications of the Swiss Mathematical Society.

Jakob Steiner (1796–1863), the great Swiss geometer, wrote this treatise on the circle and the sphere probably during the years 1823–26 while a private teacher in Berlin. The manuscript consists of 360 carefully written pages with title, book and chapter headings, evidently all prepared for immediate publication. For some reason, however, it was never published. The editors suggest