

## EVANS ON MATHEMATICAL ECONOMICS

*Mathematical Introduction to Economics.* By Griffith C. Evans. New York, McGraw-Hill Book Co., 1930. xi+177 pp. \$3.00.

As the author remarks, this book is not a voluminous or complete treatise on mathematical economics, but is a short unified account of a sequence of economic problems by means of a few rather simple mathematical methods. Undoubtedly both the mathematician who knows little or no class-room economics and the economist who has only a knowledge of the integral and differential calculus can read this book with considerable profit.

In Chapters 14 and 15, Professor Evans gives a very short account of his own economic researches and those of other recent writers on dynamical economics. Professor Evans treatment of Divisia's instantaneous price index in Chapter 9, and his application of it to economic crises, is new and is not to be found elsewhere. His treatment of units and dimension theory in Chapter 2 is unusually good. Mathematicians will likely find these chapters most stimulating but all chapters are sufficiently mathematical to hold the interest of nearly any mathematician. The text is well suited for a course to follow the traditional course in differential and integral calculus. Such mathematical concepts as maxima and minima of functions and of integrals, elementary ordinary, partial and Pfaffian differential equations, and integral equations are employed in a way which a beginner can grasp. The teacher will, however, occasionally find himself called upon for more or less detailed explanations which the author omits. For example, without previous mention of tensors the author gives the following footnote on page 20: "Wealth may perhaps be regarded as a tensor or complex number, with one component for each kind of wealth; but the concept has not a great deal of significance, since it lacks application."

Instead of following the classical arrangement of material, that is, value, utility, marginal utility, etc., and losing the reader at the outset, Professor Evans prefers to begin with more or less popular conceptions of cost of production and demand and to postpone an involved study of demand by the use of utility functions to Chapters 11 and 12. The theory of production is considered in Chapter 13. The reader should supplement this treatment of production by the recent papers of Henry Schultz.

It is in Chapter 10 that Professor Evans explains his point of view with reference to the place of economic theory. He says, "Let us admit that the entire economic aspect of human affairs is necessarily too vast to be covered by a single theory. Our endeavor then should be to make systematic study of several groups of economic situations, as theoretical investigations, and bring out the respective hypotheses which separate these groups." The "bases of action" of individuals are various. "Sometimes there is an attempt to unify them by saying that a man tries to act in such a way as to increase his pleasure. But from this point of view we have to consider at the same time not only both capitalists and laborer, but also the profiteer and the soldier, the adventurer and the hermit, the teacher, the beggar and the thief."

The first chapter is, as labelled, an elementary theory of monopoly. Other subjects treated are monopoly, change of units, competition and cooperation of producers for the cases of fixed and variable prices, diversification of cost, tariff, rent, rates of exchange, theory of interest, and the equation of exchange.

In the discussion of competition of  $n$  producers, Professor Evans treats only the case for which all producers in the same market sell their products at the same price. We know from experience that this seldom happens. It is the reviewer's opinion that it is just about impossible to give a satisfactory treatment of the static problem of competition. One could write the  $n$  prices as functions of the  $n$  quantities produced. A theory of competition could then be built up for the case for which the operations of the producers were independent of each other, that is, the Jacobian not equal to zero, and for the case for which certain groups worked together, that is, the case for which the Jacobian vanishes. The large number of such price relations would of course materially complicate the problem. Professor Harold Hotelling has given a mathematical treatment to an important aspect of static equilibrium. He points out that there are groups of buyers associated with each seller who will deal with him in preference to others in spite of a moderate price difference which may vary continuously among buyers. In the case of the recent calculus of variations treatment of the dynamic problem of competition the difficulty concerning a single price seems to be obviated. Here it is proposed that customers compare the price of any competitor with an average price for the market,  $p(t)$ , where  $t$  represents time, and if the difference is sufficiently small make the purchase without further consideration. If all prices in the same market should remain constant, it might be reasonable to suppose that after a time all customers would go to the producer who maintained the lowest price (service, etc., of course, included); but when prices change as we know they do, the producer who has the high price today may have the low price of tomorrow. In other words prices are continually changing and the prices of all successful producers of an identical commodity in the same market fall within statistical limits of an ideal theoretical price function of the time.

It would seem that the author could have begun with an introduction of the concept of profit over an interval of time as represented by an integral. His mathematical tool would then have been the calculus of variations. The Euler-Lagrange equations of the calculus of variations would have been his own conditions for maximum and minimum without alteration, and by this single stroke he could have lifted the first six chapters from the unreal world of static economics to the promised land of economic dynamics. His omission is pardonable since this procedure might have had the effect of frightening many economists and others whose mathematical knowledge ends abruptly with the calculus, but the book seems hardly complete without a concluding chapter which indicates how this transformation could be easily accomplished.

Most of the misprints noted by the reviewer will be readily recognized by the reader. The following correction, however, is necessary to avoid confusion: page 12, last sentence of first paragraph of Exercise 11 should read: "show that if an advantage of profit is possible with a given expense of advertising  $z$ , it will be increased disproportionately by increasing that expense."

C. F. Roos