

*Aufgabensammlung zur Funktionentheorie.* By Konrad Knopp. Volume II: *Aufgaben zur höheren Funktionentheorie.* Sammlung Göschen, No. 878. Berlin and Leipzig, Walter de Gruyter, 1928. 143 pp.

This little book is the continuation of Volume I (*Aufgaben zur elementaren Funktionentheorie*, No. 877) which appeared under the same title in 1923. Like all other books published by Dr. Knopp, large as well as small ones, the present volume is a new contribution, small, but significant, to the widely known reputation of its author as an excellent scholar and talented teacher. The difficult task of selecting from the immense material of the modern theory of functions the problems just within the reach of a beginner is here masterfully accomplished. A student who will go faithfully through 183 problems of Vol. I and 187 problems of Vol. II will almost imperceptibly find himself in possession of a technique which is absolutely necessary for starting an advanced study; at the same time, he will acquire general information as to the present situation of the subject. He will find problems (of different grades of difficulty) on power- and Laurent-series, on singular points of analytic functions and calculus of residues, on factorization and partial fraction expansion of entire and meromorphic functions, on simple and double periodic functions, on analytic continuation and behavior of power series on their circles of convergence, on Riemann surfaces and conformal transformations. The solutions which follow the problems are condensed enough to be valuable even to students who use them. It is the reviewer's opinion that the two volumes of the *Aufgabensammlung* together with the two volumes of Knopp's *Funktionentheorie* (Sammlung Göschen, Nos. 668, 703) can be successfully used in this country in teaching the first course of the theory of functions of a complex variable.

J. D. TAMARKIN

*L'Évolution des Idées Géométriques dans la Pensée Grecque: Point, Ligne, Surface.* By Federigo Enriques. Translated by Maurice Solovine. Paris, Gauthier-Villars, 1927.

This short treatise contains in outline the history of ancient geometry as a basis for an exposition of the philosophy of Greek geometry and the relation of ancient to modern thought. The merit of this tract lies in the exhibition of the philosophic movement. The historic part reveals certain omissions which would have necessitated the re-phrasing of some parts of the discussion relating to the evolution of the logic of geometry. Thus, in outlining Egyptian geometry, the author fails to point to the fact which has become evident in recent years that early Egyptian writers had advanced further than was formerly supposed and had reached results which as far as Greek testimony indicates, were not transmitted to ancient Greece. Thus the Greeks did not know of the remarkable approximation to the area of the circle, contained in the Rhind papyrus. More striking yet, is the content of another Egyptian papyrus (the Moscow papyrus) presumably of equal antiquity, which, like a flash of lightning in the darkness, momentarily illumines the obscure past. It contains the correct computation of the volume of the frustrum of a square pyramid. Had Enriques had in mind

this startling achievement, his discussion of the origin of the theorem relating to the volume of a pyramid would have been somewhat different. His discussion of the critique of Pythagorean concepts of points due to Parmenides and Zeno of Elea is excellent, as is also his account of the development of the infinitesimal analysis of Democritus and Archimedes, and his comparison of it with modern views. There is another historical point to which it may be worth while to direct attention. Bertrand Russell and A. N. Whitehead refer to Weierstrass as the first to banish the fixed infinitesimal from the differential and integral calculus. Enriques mentions Weierstrass, but also Cauchy and Dini. We wish to remind the reader of the historical fact that the fixed infinitesimal was banished, in the works of Benjamin Robins in 1735, of Colin Maclaurin in 1742, and of Simon Lhuillier in 1786, though, of course, these men had not reached the arithmetization of the theory of limits of the time of Weierstrass.

FLORIAN CAJORI

*Invariants of Quadratic Differential Forms.* By O. Veblen. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 24.) Cambridge University Press, 1927. viii+102 pp.

This book has been prepared to replace Tract No. 9 of the same series, *Invariants of Quadratic Differential Forms*, by J. E. Wright, which has been out of print for a number of years.

As pointed out in the preface, the presentation is formal in character as the space would permit of only the simplest applications. In Chapter I certain preliminaries are taken up such as the notation, the Kronecker deltas and then application to theorems on determinants. Chapter II is entitled *Differential Invariants*. Here the author considers such concepts as *n-dimensional space, coordinate system, invariant, tensor*, in a manner which commends the highest praise. Nowhere else have I seen these ideas so carefully presented. Chapter III considers quadratic differential forms and the theory of covariant differentiation and the curvature tensor. Chapter IV is devoted to euclidean geometry. The development is carried through largely for an *n-dimensional space* and in terms of a coordinate system not assumed to be Cartesian. In this fashion the expressions for a number of the metrical invariants of Riemannian geometry are introduced. Chapter V is given to a study of the problem of Christoffel concerning the equivalence of two quadratic differential forms. Chapter VI deals principally with the geometry of paths through the powerful method of "normal coordinates." A short historical section appears at the end of each chapter which serves to point out the chief sources of material.

In the opinion of the reviewer this book is exceedingly well done. The careful formulation of the underlying concepts and the meticulous phrasing of the definitions make this little book an invaluable one to a beginner in the subject, and for the same reason, it seems to me it must afford the adept a high degree of aesthetic enjoyment.

J. H. TAYLOR