SHORTER NOTICES

*Modern Researches on the Singularities of Functions Defined by Taylor's
   Series.* By S. Mandelbrojt. Edited by E. R. C. Miles. The Rice In-
   stitute Pamphlet, Volume XIV, No. 4, pp. 225–352.

The material collected in this pamphlet is a result of a series of lectures
delivered by the author at the Rice Institute during the academic year
1926–27. The purpose and scope are best described by a short paragraph
taken from a foot-note on the first page of the book:

"The present type of research, which began with the famous thesis of
Hadamard, has broadened considerably during the past few years. Com-
plete proofs of all the theorems cannot be compressed in the space available
in a short treatise, so the author treats only of such theorems as seem to
give unity to the theory, and of the latter theorems proofs are given, for
the greater part, in detail. The reader will find a large bibliography, as well
as an enumeration of nearly all the results on the singularities of Taylor's
series in Hadamard and Mandelbrojt: *La Série de Taylor et son Prolonge-
ment Analytique*, Scientia, No. 41, also in the author's volume of the
Mémorial des Sciences Mathématiques. The present treatise may be re-
garded as complementary to, as well as an elaboration of, some parts of
the works just mentioned."

The first four chapters deal with that part of the theory which has now
become classical, treating such material as the theorems of Hadamard and
Hurwitz on the composition of singularities and a necessary and sufficient
condition that a function should be meromorphic on its circle of conver-
gence. In Chapters V and VI the author launches into the more recent
developments of the theory, proving first an interesting theorem of his
own regarding power series which have on their circle of convergence at
least one singularity not a pole, and then giving various generalizations and
applications. Chapter VI again returns to classical theory, presenting the
theory of order of singularities. In criticism of this chapter we call atten-
tion to the fact that although the first section defines and discusses frac-
tional differentiation and integration, no mention is made of its connection
with the Hadamard operator defined in the second section. A reader meet-
ing the subject for the first time (and it is believed that the book will in
general be valuable for such readers) may naturally inquire why fractional
differentiation is introduced at all. It is not until the close of the chapter,
twenty pages farther along that one reads the statement, without proof,
that if one of these operators yields a continuous function of finite deviation
so too does the other. Chapters VIII and IX deal with the fundamental
theorems of Faber and Fatou and with the author's generalizations of the
latter. Chapters X and XI center around Dr. Mandelbrojt's contributions
to the theory and related theorems. The final chapter deals with series
having the circle of convergence as a cut, following the method introduced
by Hadamard and amplified by Fabry, Leau and others.

The reviewer believes that the book will be a welcome addition to the treatises on the subject. Mr. Miles and those responsbile for the Pamphlet are to be congratulated upon the careful way in which the lectures are edited. Misprints are rare, and the general high character of the Pamphlet, established in earlier numbers dealing with mathematical subjects, is maintained. The consistency with which references are given to unproved theorems and to the proofs of facts employed in the demonstrations is highly commendable. If this principle were followed more generally, mathematical literature would be more readable.

D. V. WIDDER

Theory of Probability. By William Burnside. Cambridge University Press, 1928. 106+xxx pp. Short table of Error-Integral.

This *Theory of Probability* was edited for the press by A. R. Forsyth from a manuscript which Burnside practically completed before his death. In barely a hundred pages, it treats a great variety of topics in a decidedly unique and interesting manner. It is not a book on statistics. Burnside's interest is in probability itself. First, he develops in somewhat extended form relations between probabilities of a general nature associated with n conditions, some of which are to be satisfied and some not—connecting his primal idea of equal likelihood with these conditions. Then he proceeds to the discussion of typical problems, mainly algebraic in their origin. Analysis is freely used—of course, it is indispensible—but the reader gets the impression that Burnside tries to view a problem first in an algebraic setting. Thus the difference equation is more in evidence than the differential equation. This is a distinctive feature. It makes the book of special value to readers who have cultivated mainly the analytic side.

The book is fairly well organized, in spite of the fact that Burnside may not have considered it in its final form. A greater unification of topics involving the Gaussian law would have been desirable,—this law appears on pages 42, 44, 52, 73, 88. On page 73, the validity of this law as the best approximation is not so clear, as the elementary probability designated by x is made very small—this leading naturally to the Poisson Exponential, obtained on page 45. Burnside is not satisfied with the usual statement of the assumption of equal likelihood. He insists upon "assuming each two of the n results equally likely," instead of "assuming all the n results to be equally likely," see page 101. Burnside's phrasing appears a little more specific—although it seems difficult to conceive how all can be equally likely if some two of the results are not equally likely. In Chapter VII, which deals with the theory of errors, assumptions are given which lead respectively to the arithmetic mean, the median, and the average of the least and the greatest measurement. The arithmetic mean is found acceptable "as any other assumption would imply that either an excess of positive errors or an excess of negative errors has occurred." If here by "positive errors" the *number* of positive errors is meant, the median would be indicated rather than the arithmetic mean. The argument in Chapter VI follows conventional lines, including an artificial assumption regarding the a priori