COURNOT ON MATHEMATICAL ECONOMICS

Researches into the Mathematical Principles of the Theory of Wealth. By Augustin Cournot, 1838. Translated by Nathaniel T. Bacon, with an Essay on *Cournot and Mathematical Economics* and a *Bibliography of Mathematical Economics*, by Irving Fisher. New York, The Macmillan Company, 1927. xxiv+213 pp.

Marshall, in his celebrated treatise,* gives a remarkable estimate of two economists of the early part of the nineteenth century. Everywhere he pays tribute to Ricardo's skill in deductive reasoning and his power of abstraction in making hypotheses. "Ricardo himself had no mathematical training. But his instincts were unique; and very few trained mathematicians could tread as safely as he over the most perilous courses of reasoning. Even the acute logical mind of Mill was unequal to the task" (p. 836). And of Cournot he writes: " · · · Cournot's genius must give a new mental activity to everyone who passes through his hands · · · " (p. xi) and "In Cournot it [the French school] has had a constructive thinker of the highest genius" (p. 766). Ricardo's work, to a mathematician, suggests the additional clarity and completeness which could be obtained by means of mathematical formulation; and Cournot, in the book under review, actually carries out a systematic mathematical discussion of certain important questions in economics "rather to present a few new views than to arrange truths already sufficiently known" (p. 164). The reviewer is able to add his testimony that from Ricardo he received the inspiration to a mathematical study of economics, and in this book of Cournot,† which he still regards as the best elementary treatise on the theory of economics, found such a study already well developed.

This is not to say that the book is perfect. In particular, the last chapter, on *Variation in the Social Income, Resulting from the Communication of Markets,* seems not to be adequate in its analysis, for changes in the *form* of production, resulting from such communication, are not carefully considered. And there are occasional slips, usually not of serious character, which in this edition are noted or corrected. Nevertheless the reviewer maintains his opinion that the reader of inquiring mind will get more out of this book than from any other yet published on the subject.

Moreover Cournot is almost alone in holding to a clear realization of the difference between measurable and non-measurable quantities, pointing out in Chapter 1 that many of the controversies in the subject arise from ambiguities of this sort. In the next to the last chapter this clearness is manifest, for the author is able to build up a serviceable theory of the "Social Income," on the basis of money value.

On the other hand, one recent book on the mathematical principles of

* *Principles of Economics*, London, 1920.

† The first French edition, in the Harvard College Library.

economics, typical of many others, builds its theory on the following basis.*

"Write $U(x, y, \cdots)$ for an algebraic function of measurable quantities $x, y \cdots$; let it be so related to an entity we will call $S(x, y, \cdots)$, where S is not a calculable function but the non-measurable satisfaction derived from quantities $x, y, \cdots$, that the following postulates are satisfied.

"*Postulates.* (1) When $x, y, \cdots$ vary without affecting the value of $U(x, y, \cdots)$, more x balancing less y, etc., $S(x, y, \cdots)$ remains unchanged.

"(2) When $x, y, \cdots$ vary so as to increase $U(x, y, \cdots)$, $S(x,y, \cdots)$ increases, and if U decreases, S decreases.

"(3) When there are successive variations of $x, y, \cdots$, the first increasing U from U_1 to U_2, the second from U_2 to U_3, so that the second increase is greater than the first $(U_3 - U_2 > U_2 - U_1)$, then the second increase in S is greater than the first; the postulate still to be true when less is written for greater."

Apparently this other author is unaware that he is begging the question. If loci of indifference are expressed by Pfaffian differential equations it does not follow that there is any function of which these are the level loci, for such equations are not necessarily completely integrable. The question is not of names, but of existence. These supposedly general treatments are much more special than their authors imagine.†

To carry the point further, we suggest that most nouns that denote values may be regarded as misnomers. They have possibly a significance "im kleinen"—a local significance,—derived from the local integration of the equation, but no significance in extenso. Not only in ethics and aesthetics would an adjective be "true-er" than a noun; it may be presumptuous even to insist, in the words of a recent controversy on a point in the teaching of the calculus, "that there is only one truth." With that remark, and the advice to economists to follow Cournot, rather than Jevons and his school, in this respect, these problems may well be left to the more experienced consideration of philosophers.

Besides the general problems discussed in the first two and last two chapters, to which we have already alluded, Cournot's book deals in considerable detail with the subjects of foreign exchange, law of demand, monopoly, competition, competition among a large number of producers, producers who furnish parts of a compound product, and the communication of markets. The effects of taxation are investigated with respect to the various forms of production.

Throughout, the problems are analyzed as far as possible in terms of functions of a single variable. Generalizations in this respect are important, but not so difficult for a mathematician to make as for the reader to follow, if he is of the type for which this book was intended. Such generalizations, among other contributions, have been made by Walras and Pareto. Still more significant extensions would arise from the abandonment of the hypothesis, which Cournot and almost all other economists make

* Bowley, *Mathematical Groundwork of Economics*, Oxford, 1924, p. 1.

† The trouble usually comes from reasoning about n dimensions with the intuition merely of a two-dimensional figure.

either explicitly or tacitly, that demand is a function of price, adequately represented by the notation $D=f(p)$. Cournot is also not alone in considering profit as the central quantity in economics, whose maximum determines the other quantities; and given the industrial habits of his time, he is hardly to be blamed.

The translation seems to be carefully done, and the editor has added numerous notes designed to assist the less mathematical reader. No attempt has been made to bring the bibliography down to the date of this edition; in the thirty years since the treatise first appeared in English the additions to the bibliography included in that edition would presumably have surpassed the desirable limits for the book. There would have been service rendered, however, if the editor had used the resources at his disposal to clear up the ambiguous character of some of the references already given. This is not said to diminish the gratitude which the public owes to both editor and translator in making again accessible, after ninety years, the eternal freshness of this brief classic.

G. C. EVANS

THREE BOOKS ON NON-EUCLIDEAN GEOMETRY

Vorlesungen über Nicht-Euklidische Geometrie. By Felix Klein. Prepared anew for publication by W. Rosemann. Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, vol. 26. Berlin, Julius Springer, 1928. xii+326 pp. Price 18 marks unbound, 19.50 marks in linen.

Nicht-Euklidische Geometrie. Hyperbolische Geometrie der Ebene. By Richard Baldus. Sammlung Göschen, No. 970. Berlin and Leipzig, Walter de Gruyter, 1927. 152 pp. Price 1.50 marks.

La Géométrie Non-Euclidienne. By P. Barbarin. Third edition, followed by notes by A. Buhl. Collection "Scientia," No. 15. Paris, Gauthier-Villars, 1928. 176 pp.

In a note to his Erlanger Programm, Klein justified his avoidance of the word "non-euclidean" in that address, saying: "There are connected with the designation 'non-euclidean geometry' a number of unmathematical conceptions which occasion as much enthusiasm on the one side as horror on the other, but which have nothing whatever to do with our purely mathematical considerations." This may be a reason why, aside from a few glances forward, that same word is absent from over one half of his admirable treatise, *Vorlesungen über Nicht-Euklidische Geometrie.* In fact, the reader with a purely euclidean past will be led so tactfully and persuasively from his usual geometry to the more general one that he may long wonder wherein lies the book's right to its title. He will be an admirer of the greater beauty and symmetry of non-euclidean space without realizing how far he has left euclidean space behind him.

Part one of the book is an introduction to projective geometry. We pass rapidly from elementary affine coordinates in euclidean space to homogeneous and general projective coordinates. Thus, the arithmetical basis