ON THE MAXIMUM DEFICIENCY OF r-SPACE CURVES

BY B. C. WONG

The problem of determining the greatest deficiency of an r-space curve of given order seems yet unsolved. This problem is equivalent to that of determining the maximum dimension of a sub-space of S_r in which a curve of given order and genus can lie. Thus the maximum genus of a 4-space sextic curve is 2; but if the sextic curve is of genus 3 or 4, it is necessarily a 3-space curve or a plane curve; and if it is of genus greater than 4, it is a plane curve.

It is well known that the deficiency of an r-space curve C^n of order r is zero. Veronese* has shown that the greatest deficiency of an r-space curve C^{r+s} where $s<r$ is s and that of C^{2r} is $r+1$. It is the purpose of this paper to present a method of calculating the maximum deficiency of curves whose orders are greater than $2r$.

To determine the greatest deficiency P of a curve C^n, it is only necessary to determine the least number, h, of apparent double points of its projection on an S_3, for

$$p = (n-1)(n-2)/2 - h. \tag{1}$$

We shall use the phrase *the apparent double points of* C^n instead of *the apparent double points of the* 3-*space projection of* C^n. To determine h, we make use of the formula†

$$D = \mu_1\mu_2\cdots\mu_{r-1}(\mu_1\mu_2\cdots\mu_{r-1} - \sum\mu_i + r - 2)/2)\,, \tag{2}$$

for the number D of apparent double points of C^n which is the complete intersection of $r-1$ hypersurfaces of order

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* *Behandlung der projectivischen Verhältnisse der Räume von verschiedenen Dimensionen durch das Princip des Projicirens und Schneidens,* Mathematische Annalen, vol. 19 (1882), pp. 161–234.

† Loc. cit., p. 205.

$\mu_1, \mu_2, \cdots, \mu_{r-1}$. D will be a minimum if $r-2$ of the hypersurfaces are hyperquadrics. Hence, putting

$$\mu_1 = \mu_2 = \cdots = \mu_{r-2} = 2, \quad \mu_{r-1} = n/2^{r-2},$$

in (2), we have

$$h \geqq \frac{n}{2^{r-1}}[(2^{r-2} - 1)n - 2^{r-2}(r - 2)], \tag{3}$$

and

$$p \leqq \frac{1}{2^{r-1}}[n^2 + 2^{r-2}(r - 5)n + 2^{r-1}]. \tag{4}$$

The equality signs hold when $n = 2^{r-2}\,\mu_{r-1}$, where $\mu_{r-1} > 1$. When $n \neq 2^{r-2}\mu_{r-1}$, (3) gives too low a limit for h and (4) gives too high a limit for P. To find the exact value of h we distinguish two cases: n even and n odd.

First take the case n even and equal to $2m$. Consider an equivalent degenerate curve C^{2m} made up of two $(r-1)$-space curves C^m and C'^m lying in different $(r-1)$-spaces but having m points in common. If h' is the least number of apparent double points on each of C^m and C'^m, then

$$h = m^2 - m + 2h',$$

since $m^2 - m$ is the number of apparent intersections of C^m and C'^m. Replacing m by $n/2$ in the above, we have

$$h = n(n - 2)/4 + 2h' \tag{5}$$

and

$$p = (n - 2)^2/4 - 2h'. \tag{6}$$

Now take the case n odd and equal to $2m - 1$. Consider an equivalent degenerate curve C^{2m-1} composed of two $(r-1)$-space curves C^m and C^{m-1} lying in different $(r-1)$-spaces but having $m-1$ points in common. Letting h'' be the least number of apparent double points on C^m and h''' be that of apparent double points on C^{m-1}, we have

$$h = (m - 1)^2 + h'' + h''',$$

since $(m-1)^2$ is the number of apparent intersections of C^m and C^{m-1}. If we replace m by $(n+1)/2$, the result is

$$h = (n-1)^2/4 + h'' + h''' \tag{7}$$

and hence

$$P = (n-1)(n-3)/4 - h'' - h'''. \tag{8}$$

To find h' or h'' and h''' we repeat the process. We soon arrive at the component curves whose orders are equal to or less than twice the dimensions of the several sub-spaces in which the component curves lie, and then apply Veronese's rule. Or, in case n is very large, we arrive at the component curves which are all plane and have no double points.

It is to be noticed that $h'=h''=h'''=0$ if $r=3$. Hence the greatest deficiency of any 3-space curve C^n is

$$p = (n-2)^2/4, \qquad \text{for } n \text{ even};$$

and

$$p = (n-1)(n-3)/4, \qquad \text{for } n \text{ odd}.$$

THE UNIVERSITY OF CALIFORNIA

A NOTE ON CERTAIN CONTINUOUS NON-DIFFERENTIABLE FUNCTIONS*

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This note gives a treatment of some phases of the theory of a class of functions of which a particular example has already been studied by the author in a note entitled *An elementary example of a continuous non-differentiable function*, in the American Mathematical Monthly (vol. 34 (1927), pp. 476–478). The method there used for the construction of a function with the desired properties bears some resemblance to that used by Brodén, Köpke, and Steinitz for the

* Presented to the Society, September 6, 1928.