A NEW TABLE OF THE ZEROS OF THE BESSEL FUNCTIONS $J_0(x)$ AND $J_1(x)$ WITH CORRESPONDING VALUES OF $J_1(x)$ AND $J_0(x)$*

BY H. T. DAVIS AND W. J. KIRKHAM

1. *Introduction.* In connection with a problem involving the calculation of the distribution of energy in a diffraction pattern of sixty-six rings, it was found necessary to extend existing tables of the zeros of the Bessel functions $J_0(x)$ and $J_1(x)$ with corresponding values of $J_1(x)$ and $J_0(x)$. In the hope that these computations may be of use to other investigators they are reproduced here.

The first ten zeros in Table I and the first fifty zeros with the associated values of $J_0(x)$ in Table II are incorporated from tables by E. Meissel published in 1888 and 1889–90 respectively.† The values of the zeros of $J_0(x)$ and their logarithms from $s=11$ to $s=40$ have been taken from a paper by R. W. Willson and B. O. Peirce.‡ The associated values of $J_1(x)$ have been recomputed to ten decimal places and one correction in the Willson-Peirce table is noted, the value corresponding to $s=35$ being in error.

2. *Formulas Used in the Calculation of the Zeros of $J_0(x)$ and $J_1(x)$.* In computing the values of $x_0^{(s)}$ and $x_1^{(s)}$, the roots of $J_0(x)$ and $J_1(x)$ respectively, the formulas of G. C. Stokes were used.§ Thus we have

$$x_0^{(s)} = \frac{1}{4}\pi a\left(1 + \frac{\alpha_2}{a^2} + \frac{\alpha_4}{a^4} + \frac{\alpha_6}{a^6} + \frac{\alpha_8}{a^8}\right), \qquad a = 4s - 1,$$

* These tables were made possible by a grant of funds from the Waterman Institute, Indiana University.

† Mathematische Abhandlungen der Akademie der Wissenschaften zu Berlin, 1888; Annual report of the Ober-Realschule at Kiel, 1889–90.

‡ This Bulletin, vol. 3 (1897), pp. 153–155.

§ Cambridge Philosophical Transactions, vol. 9, or Mathematical and Physics Papers, vol. II, pp. 353, 355. See also J. McMahon, Annals of Mathematics, vol. 9 (1894–95), p. 25.

where

$$\alpha_2 = 2/\pi^2, \quad \alpha_4 = -\,62/(3\pi^4), \quad \alpha_6 = 15116/(15\pi^6),$$

$$\alpha_8 = -\,12554474/(105\pi^8);$$

$$x_0^{(s)} = \pi s - .7853981634 + \frac{k_1}{a} - \frac{k_2}{a^3} + \frac{k_3}{a^5} - \frac{k_4}{a^7},$$

where

$$\log k_1 = 9.2018201316, \qquad \log k_2 = 9.2217608253,$$

$$\log k_3 = 9.9155362692, \qquad \log k_4 = 0.9955001222.$$

$$x_1^{(s)} = \frac{1}{4}\pi b\left(1 + \frac{\beta_2}{b^2} + \frac{\beta_4}{b^4} + \frac{\beta_6}{b^6} + \frac{\beta_8}{b^8}\right), \qquad b = 4s + 1,$$

where

$$\beta_2 = -\,6/\pi^2, \quad \beta_4 = 6/\pi^4, \quad \beta_6 = -\,4716/(5\pi^6),$$

$$\beta_8 = 3902418/(35\pi^8);$$

$$x_1^{(s)} = \pi s + .7853981634 - \frac{m_1}{b} + \frac{m_2}{b^3} - \frac{m_3}{b^5} + \frac{m_4}{b^7},$$

where

$$\log m_1 = 9.6789413863, \qquad \log m_2 = 8.6846416409,$$

$$\log m_3 = 9.8867944373, \qquad \log m_4 = 0.9651566416.$$

The values of $x_0^{(s)}$ and $x_1^{(s)}$ were calculated from these formulas by means of Vega's ten-place table of logarithms. In checking the zeros, the differences between successive values were computed on a ten-bank Monroe calculator with the aid of Barlow's table of reciprocals and cubes from the following formulas:

$$x_0^{(s+1)} - x_0^{(s)} = \pi - .1591549431[1/(4s-1) - 1/(4s+3)] + .1666329280[1/(4s-1)^3 - 1/(4s+3)^3],$$

$$x_1^{(s+1)} - x_1^{(s)} = \pi + .4774648293[1/(4s+1) - 1/(4s+5)] - .0483773016[1/(4s+1)^3 - 1/(4s+5)^3].$$

3. *The Calculation of* $J_1[x_0^{(s)}]$ *and* $J_0[x_1^{(s)}]$. For the calculation of $J_1[x_0^{(s)}]$ and $J_0[x_1^{(s)}]$ new asymptotic formulas were used. It is well known* that $J_0(x)$ and $J_1(x)$ have the following asymptotic expansions:

$$(1)\begin{cases} J_0(x) = \left(\dfrac{2}{\pi x}\right)^{1/2}[P_0 \cos(x-\pi/4) + Q_0 \sin(x-\pi/4)], \\ J_1(x) = \left(\dfrac{2}{\pi x}\right)^{1/2}[P_1 \cos(x-\pi/4) + Q_1 \sin(x-\pi/4)], \end{cases}$$

where

$$P_0 = 1 - \frac{1^2 3^2}{2!(8x)^2} + \frac{1^2 3^2 5^2 7^2}{4!(8x)^4} - \cdots,$$

$$Q_0 = \frac{1^2}{1!8x} - \frac{1^2 3^2 5^2}{3!(8x)^3} + \frac{1^2 3^2 5^2 7^2 9^2}{5!(8x)^5} - \cdots,$$

$$P_1 = \frac{3}{1!8x} + \frac{3\cdot 5\cdot 21}{3!(8x)^3} + \frac{3\cdot 5\cdot 21\cdot 45\cdot 77}{5!(8x)^5} - \cdots,$$

$$Q_1 = 1 + \frac{3\cdot 5}{2!(8x)^2} - \frac{3\cdot 5\cdot 21\cdot 45}{4!(8x)^4} + \cdots.$$

If in the formulas (1) we let $x = x_1^{(s)}$ and use the identity $\sin^2(x-\pi/4)+\cos^2(x-\pi/4)=1$ to eliminate the sine and cosine terms, we get the following asymptotic formula for the calculation of $J_0(x_1^{(s)})$:

$$(2)\quad \begin{aligned} J_0(x_1^{(s)}) &= \pm\left(\frac{2}{\pi x_1}\right)^{1/2}\left[1 + \frac{1\cdot 3}{2!(2x_1)^2} - \frac{1^4 3^3 5}{4!(2x_1)^4} + \frac{1^4 3^4 5^3 7}{6!(2x_1)^6} - \cdots\right]^{-1/2}, \\ &= \pm\left(\frac{2}{\pi x_1}\right)^{1/2}\left[1 - \frac{3}{2^4 x_1^2} + \frac{3^2 13}{2^9 x_1^4} - \frac{3^2\cdot 5\cdot 7\cdot 23}{2^{13} x_1^6} + \frac{3^4\cdot 5\cdot 10487}{2^{19} x_1^8} + \cdots\right]. \end{aligned}$$

* See Gray, Mathews and MacRobert, *A Treatise on Bessel Functions*, London, 1922, pp. 57–59.

The sign is plus when s is even, and is minus when s is odd.

Similarly, replacing x by $x_0^{(s)}$ and eliminating the sine and cosine terms from (1) we obtain the following formula for $J_1(x_0^{(s)})$:

$$
\begin{aligned}
J_1(x_0^{(s)}) &= \pm\left(\frac{2}{\pi x_0}\right)^{1/2}\left[1-\frac{1^4}{2!(2x_0)^2}+\frac{1^4 3^4}{4!(2x_0)^4}-\frac{1^4 3^4 5^4}{6!(2x_0)^6}+\cdots\right]^{-1/2},\\
&= \pm\left(\frac{2}{\pi x_0}\right)^{1/2}\left[1+\frac{1}{2^4 x_0^2}-\frac{3\cdot 17}{2^9 x_0^4}+\frac{43\cdot 101}{2^{13}x_0^6}-\frac{3025837}{2^{19}x_0^8}+\cdots\right].
\end{aligned}
\tag{3}
$$

The sign is plus when s is odd, and is minus when s is even.

If in (2) we substitute the value of $x_1^{(s)}$ from Stokes' formula, and in (3) the value of $x_0^{(s)}$, we obtain new formulas which are remarkable in that terms involving the reciprocals of $(4s+1)^2$ and $(4s-1)^2$ are absent:

$$
J_0(x_1^{(s)}) = \pm\frac{2\cdot 2^{1/2}}{\pi b^{1/2}}\left[1+\frac{24}{\pi^4 b^4}-\frac{19584}{10\pi^6 b^6}+\frac{2466720}{7\pi^8 b^8}\right],\quad b = 4s+1,
$$

$$
J_1(x_0^{(s)}) = \pm\frac{2\cdot 2^{1/2}}{\pi a^{1/2}}\left[1-\frac{56}{3\pi^4 a^4}+\frac{9664}{5\pi^6 a^6}-\frac{7381280}{21\pi^8 a^8}\right],\quad a = 4s-1.
$$

From these formulas we see that to ten decimal places

$$
J_0(x_1^{(s)}) = \frac{2\cdot 2^{1/2}}{\pi}\,\frac{1}{(4s+1)^{1/2}} \text{ and } J_1(x_0^{(s)}) = \frac{2\cdot 2^{1/2}}{\pi}\,\frac{1}{(4s-1)^{1/2}}
$$

for values of s greater than 10. In the calculation of $J_0(x_1^{(s)})$ and $J_1(x_0^{(s)})$ from these formulas Vega's ten-place table of logarithms was used. Each value was checked by direct computation on the Monroe calculator. The first five values of $J_1(x_0^{(s)})$ were reduced from twelve-place values obtained by interpolation from Meissel's table.

4. *Formulas for the Calculation of $J_n(x)$ from Tables of $J_0(x)$ and $J_1(x)$.* It should be noted that corresponding values of $J_n(x)$ can be obtained from tables of $J_0(x)$ and $J_1(x)$ by the well known recurrence formula:

$$\frac{2nJ_n(x)}{x} = J_{n-1}(x) + J_{n+1}(x).$$

However, when n is large, the labor of calculation becomes very great. The following formulas, which, as far as the authors are aware, are not found in standard treatises on the Bessel functions, are very useful in the calculation of $J_n(x)$ from tables of $J_0(x)$ and $J_1(x)$:

$$J_{2n}(x) = (-1)^n\left[1 + \sum_{r=1}^{n-1}(-1)^r \frac{2^{2r}n^2(n^2-1)^2\cdots}{(2r)!x^{2r}} \cdot \frac{[n^2-(r-1)^2]^2(n^2-r^2)}{1}\right]J_0(x)$$
$$+ (-1)^{n+1}\left[\sum_{r=0}^{n-1}(-1)^r \frac{2^{2r+1}n^2(n^2-1)^2(n^2-4)^2}{(2r+1)!x^{2r+1}} \cdot \frac{\cdots(n^2-r^2)^2}{1}\right]J_1(x).$$

$$J_{2n-1}(x) = (-1)^{n+1}\left[\sum_{r=0}^{n-2}(-1)^r \frac{2^{2r+1}n^2(n^2-1)^2\cdots}{(2r+1)!x^{2r+1}} \cdot \frac{[n^2-(r-1)^2]^2(n^2-r^2)(n-r)(n-r-1)}{1}\right]J_0(x)$$
$$+ (-1)^{n+1}\left[1 + \sum_{r=1}^{n-1}(-1)^r \frac{2^{2r}n^2(n^2-1)^2\cdots}{(2r)!x^{2r}} \cdot \frac{[n^2-(r-1)^2]^2(n-r)^2}{1}\right]J_1(x).$$

These formulas are easily proved by induction.

TABLE I.

THE FIRST 150 ROOTS OF $J_0(x)$ WITH CORRESPONDING VALUES OF $J_1(x)$

s	$x_0^{(s)}$	$\log x_0^{(s)}$	$J_1[x_0^{(s)}]$	$\log \pm J_1[x_0^{(s)}]$
1	2.4048255577	0.3810835788	+0.5191474973	9.715290765
2	5.5200781103	0.7419452231	−0.3402648065	9.531817033
3	8.6537279129	0.9372032361	+0.2714522999	9.433693526
4	11.7915344391	1.0715703238	−0.2324598214	9.366347900
5	14.9309177086	1.1740865018	+0.2065464331	9.315017699
6	18.0710639679	1.2569837232	−0.1877288030	9.273530911
7	21.2116366299	1.3265741787	+0.1732658943	9.238713084
8	24.3524715308	1.3865430443	−0.1617015504	9.208714184
9	27.4934791320	1.4392297006	+0.1521812139	9.182361044
10	30.6346064684	1.4862123057	−0.1441659779	9.158862782
11	33.7758202136	1.5286059043	+0.1372969435	9.137660869
12	36.9170983537	1.5672275586	−0.1313246267	9.118346175
13	40.0584257646	1.6026938781	+0.1260694971	9.100610020
14	43.1997917132	1.6354816528	−0.1213986249	9.084213767
15	46.3411883717	1.6659671666	+0.1172111989	9.068969108
16	49.4826098974	1.6944525978	−0.1134291926	9.054724841
17	52.6240518411	1.7211842839	+0.1099911432	9.041357716
18	55.7655107550	1.7463656842	−0.1068478884	9.028765943
19	58.9069839261	1.7701667872	+0.1039595729	9.016864487
20	62.0484691902	1.7927310714	−0.1012934991	9.005581573
21	65.1899648002	1.8141807465	+0.0988225538	8.994856073
22	68.3314693299	1.8346207594	−0.0965240405	8.984635493
23	71.4729816036	1.8541418997	+0.0943787940	8.974874423
24	74.6145006437	1.8728232368	−0.0923705049	8.965533317
25	77.7560256304	1.8907340543	+0.0904851942	8.956577523
26	80.8975558711	1.9079354006	−0.0887108024	8.947976507
27	84.0390907769	1.9244813451	+0.0870368634	8.939703232
28	87.1806298436	1.9404200023	−0.0854542429	8.931733631
29	90.3221726372	1.9557943757	+0.0839549293	8.924046200
30	93.4637187819	1.9706430570	−0.0825318613	8.916621640
31	96.6052679510	1.9850008094	+0.0811787883	8.909442565
32	99.7468198587	1.9988990584	−0.0798901543	8.902493260
33	102.8883742542	2.0123663047	+0.0786610017	8.895759473
34	106.0299309165	2.0254284784	−0.0774868911	8.889228237
35	109.1714896498	2.0381092361	+0.0763638333	8.882887721
36	112.3130502805	2.0504302219	−0.0752882326	8.876727102
37	115.4546126537	2.0624112882	+0.0742568382	8.870736454
38	118.5961766309	2.0740706879	−0.0732667027	8.864906647
39	121.7377420880	2.0854252422	+0.0723151467	8.859229272
40	124.8793089132	2.0964904866	−0.0713997282	8.853696559

TABLE I. (Continued)

s	$x_0^{(s)}$	$\log x_0^{(s)}$	$J_1[x_0^{(s)}]$	$\log \pm J_1[x_0^{(s)}]$
41	128.0208770059	2.1072807979	+0.0705182163	8.848301319
42	131.1624462752	2.1178095078	−0.0696685682	8.843036885
43	134.3040166383	2.1280890011	+0.0688489095	8.837897066
44	137.4455880203	2.1381308034	−0.0680575164	8.832876097
45	140.5871603528	2.1479456587	+0.0672928009	8.827968605
46	143.7287335737	2.1575435988	−0.0665532972	8.823169576
47	146.8703076258	2.1669340045	+0.0658376495	8.818474318
48	150.0118824570	2.1761256608	−0.0651446023	8.813878437
49	153.1534580192	2.1851268069	+0.0644729905	8.809377815
50	156.2950342685	2.1939451798	−0.0638217315	8.804968583
51	159.4366111643	2.2025880546	+0.0631898176	8.800647102
52	162.5781886689	2.2110622805	−0.0625763097	8.796409948
53	165.7197667480	2.2193743133	+0.0619803313	8.792253893
54	168.8613453692	2.2275302450	−0.0614010632	8.788175891
55	172.0029245031	2.2355358310	+0.0608377387	8.784173063
56	175.1445041219	2.2433965140	−0.0602896398	8.780242689
57	178.2860842001	2.2511174463	+0.0597560927	8.776382192
58	181.4276647137	2.2587035104	−0.0592364646	8.772589131
59	184.5692456406	2.2661593371	+0.0587301608	8.768861190
60	187.7108269600	2.2734893229	−0.0582366213	8.765196170
61	190.8524086526	2.2806976452	+0.0577553186	8.761591984
62	193.9939907001	2.2877882769	−0.0572857554	8.758046644
63	197.1355730857	2.2947649996	+0.0568274620	8.754558260
64	200.2771557933	2.3016314151	−0.0563799947	8.751125031
65	203.4187388082	2.3083909573	+0.0559429339	8.747745239
66	206.5603221162	2.3150469021	−0.0555158823	8.744417247
67	209.7019057043	2.3216023771	+0.0550984638	8.741139490
68	212.8434895599	2.3280603705	−0.0546903214	8.737910475
69	215.9850736715	2.3344237388	+0.0542911166	8.734728774
70	219.1266580280	2.3406952151	−0.0539005280	8.731593019
71	222.2682426191	2.3468774157	+0.0535182499	8.728501903
72	225.4098274349	2.3529728465	−0.0531439918	8.725454173
73	228.5514124661	2.3589839095	+0.0527774772	8.722448626
74	231.6929977040	2.3649129085	−0.0524184425	8.719484113
75	234.8345831404	2.3707620540	+0.0520666369	8.716559527
76	237.9761687673	2.3765334684	−0.0517218210	8.713673807
77	241.1177545773	2.3822291905	+0.0513837662	8.710825933
78	244.2593405633	2.3878511803	−0.0510522546	8.708014926
79	247.4009267187	2.3934013221	+0.0507270777	8.705239844
80	250.5425130370	2.3988814292	−0.0504080363	8.702499779

TABLE I. (Continued)

s	$x_0^{(s)}$	$\log x_0^{(s)}$	$J_1[x_0^{(s)}]$	$\log \pm J_1[x_0^{(s)}]$
81	253.6840995122	2.4042932472	+0.0500949399	8.699793860
82	256.8256861386	2.4096384570	−0.0497876061	8.697121244
83	259.9672729106	2.4149186784	+0.0494858602	8.694481124
84	263.1088598231	2.4201354726	−0.0491895350	8.691872717
85	266.2504468710	2.4252903453	+0.0488984701	8.689295272
86	269.3920340498	2.4303847494	−0.0486125117	8.686748061
87	272.5336213547	2.4354200870	+0.0483315122	8.684230384
88	275.6752087815	2.4403977121	−0.0480553299	8.681741563
89	278.8167963262	2.4453189327	+0.0477838286	8.679280944
90	281.9583839846	2.4501850127	−0.0475168778	8.676847897
91	285.0999717532	2.4549971743	+0.0472543516	8.674441808
92	288.2415596282	2.4597565990	−0.0469961292	8.672062089
93	291.3831476063	2.4644644304	+0.0467420942	8.669708166
94	294.5247356841	2.4691217748	−0.0464921346	8.667379487
95	297.6663238584	2.4737297040	+0.0462461428	8.665075516
96	300.8079121264	2.4782892553	−0.0460040147	8.662795734
97	303.9495004850	2.4828014340	+0.0457656504	8.660539638
98	307.0910889315	2.4872672144	−0.0455309532	8.658306742
99	310.2326774632	2.4916875409	+0.0452998301	8.656096573
100	313.3742660776	2.4960633298	−0.0450721913	8.653908673
101	316.5158547720	2.5003954693	+0.0448479502	8.651742598
102	319.6574435444	2.5046848218	−0.0446270230	8.649597916
103	322.7990323923	2.5089322243	+0.0444093289	8.647474210
104	325.9406213135	2.5131384890	−0.0441947898	8.645371073
105	329.0822103059	2.5173044056	+0.0439833303	8.643288109
106	332.2237993677	2.5214307406	−0.0437748773	8.641224937
107	335.3653884968	2.5255182391	+0.0435693603	8.639181183
108	338.5069776912	2.5295676252	−0.0433667110	8.637156486
109	341.6485669493	2.5335796034	+0.0431668633	8.635150492
110	344.7901562692	2.5375548582	−0.0429697534	8.633162861
111	347.9317456494	2.5414940560	+0.0427753191	8.631193258
112	351.0733350881	2.5453978449	−0.0425835005	8.629241359
113	354.2149245839	2.5492668559	+0.0423942396	8.627306850
114	357.3565141352	2.5531017031	−0.0422074799	8.625389422
115	360.4981037404	2.5569029846	+0.0420231669	8.623488778
116	363.6396933985	2.5606712829	−0.0418412476	8.621604625
117	366.7812831078	2.5644071655	+0.0416616706	8.619736681
118	369.9228728672	2.5681111852	−0.0414843861	8.617884667
119	373.0644626753	2.5717838810	+0.0413093457	8.616048316
120	376.2060525309	2.5754257783	−0.0411365025	8.614227364

TABLE I. (Continued)

s	$x_0^{(s)}$	$\log x_0^{(s)}$	$J_1[x_0^{(s)}]$	$\log \pm J_1[x_0^{(s)}]$
121	379.3476424328	2.5790373893	+0.0409658109	8.612421556
122	382.4892323800	2.5826192136	−0.0407972266	8.610630640
123	385.6308223712	2.5861717385	+0.0406307066	8.608854375
124	388.7724124055	2.5896954395	−0.0404662091	8.607092521
125	391.9140024818	2.5931907804	+0.0403036936	8.605344848
126	395.0555925990	2.5966582142	−0.0401431204	8.603611128
127	398.1971827563	2.6000981830	+0.0399844514	8.601891141
128	401.3387729527	2.6035111185	−0.0398276490	8.600184671
129	404.4803631872	2.6068974423	+0.0396726770	8.598491506
130	407.6219534590	2.6102575659	−0.0395195001	8.596811442
131	410.7635437673	2.6135918920	+0.0393680838	8.595144276
132	413.9051341111	2.6169008134	−0.0392183947	8.593489813
133	417.0467244898	2.6201847145	+0.0390704003	8.591847860
134	420.1883149024	2.6234439708	−0.0389240687	8.590218230
135	423.3299053483	2.6266789495	+0.0387793690	8.588600738
136	426.4714958268	2.6298900094	−0.0386362712	8.586995206
137	429.6130863371	2.6330775018	+0.0384947459	8.585401458
138	432.7546768784	2.6362417700	−0.0383547646	8.583819321
139	435.8962674503	2.6393831501	+0.0382162993	8.582248629
140	439.0378580519	2.6425019709	−0.0380793229	8.580689217
141	442.1794486827	2.6455985539	+0.0379438088	8.579140923
142	445.3210393420	2.6486732140	−0.0378097313	8.577603591
143	448.4626300292	2.6517262595	+0.0376770652	8.576077067
144	451.6042207439	2.6547579922	−0.0375457858	8.574561198
145	454.7458114852	2.6577687076	+0.0374158692	8.573055839
146	457.8874022529	2.6607586950	−0.0372872920	8.571560843
147	461.0289930463	2.6637282380	+0.0371600312	8.570076070
148	464.1705838648	2.6666776142	−0.0370340647	8.568601380
149	467.3121747080	2.6696070956	+0.0369093705	8.567136638
150	470.4537655754	2.6725169490	−0.0367859274	8.565681710

TABLE II.

THE FIRST 150 ROOTS OF $J_1(x)$ WITH CORRESPONDING VALUES OF $J_0(x)$

s	$x_1^{(s)}$	$\log x_1^{(s)}$	$J_0[x_1^{(s)}]$	$\log \pm J_0[x_1^{(s)}]$
1	3.8317059702	0.5833921757	−0.4027593957	9.605045681
2	7.0155866698	0.8460639942	+0.3001157525	9.477288791
3	10.1734681351	1.0074690286	−0.2497048771	9.397427025
4	13.3236919363	1.1246245823	+0.2183594072	9.339171907
5	16.4706300509	1.2167102124	−0.1964653715	9.293286014
6	19.6158585105	1.2926073202	+0.1800633753	9.255425387
7	22.7600843806	1.3571738678	−0.1671846005	9.223196271
8	25.9036720876	1.4133613337	+0.1567249863	9.195138240
9	29.0468285340	1.4630987210	−0.1480111100	9.170294315
10	32.1896799110	1.5077166581	+0.1406057982	9.148003230
11	35.3323075501	1.5481720021	−0.1342112403	9.127788890
12	38.4747662348	1.5851759898	+0.1286166221	9.109297099
13	41.6170942128	1.6192717536	−0.1236679608	9.092257199
14	44.7593189977	1.6508834702	+0.1192498120	9.076457703
15	47.9014608872	1.6803487586	−0.1152736941	9.061730211
16	51.0435351836	1.7079407452	+0.1116704969	9.047938448
17	54.1855536411	1.7338835151	−0.1083853489	9.034970580
18	57.3275254379	1.7583631957	+0.1053740554	9.022733694
19	60.4694578453	1.7815360748	−0.1026005671	9.011149761
20	63.6113566985	1.8035346583	+0.1000351468	9.000152613
21	66.7532267341	1.8244722636	−0.0976530158	8.989685660
22	69.8950718375	1.8444465556	+0.0954333390	8.979700119
23	73.0368952256	1.8635423032	−0.0933584533	8.970153648
24	76.1786995846	1.8818335547	+0.0914132722	8.961009255
25	79.3204871755	1.8993853729	−0.0895848220	8.952234435
26	82.4622599144	1.9162552327	+0.0878618760	8.943800472
27	85.6040194364	1.9324941569	−0.0862346634	8.935681873
28	88.7457671449	1.9481476480	+0.0846946348	8.927855900
29	91.8875042517	1.9632564558	−0.0832342730	8.920302190
30	95.0292318080	1.9778572186	+0.0818469379	8.913002436
31	98.1709507308	1.9919829969	−0.0805267394	8.905940115
32	101.3126618230	2.0056637255	+0.0792684317	8.899100266
33	104.4543657913	2.0189265960	−0.0780673254	8.892469301
34	107.5960632595	2.0317963811	+0.0769192140	8.886034837
35	110.7377547809	2.0442957136	−0.0758203116	8.879785565
36	113.8794408476	2.0564453260	+0.0747672005	8.873711120
37	117.0211218989	2.0682642573	−0.0737567865	8.867801987
38	120.1627983281	2.0797700333	+0.0727862602	8.862049406
39	123.3044704886	2.0909788222	−0.0718530644	8.856445295
40	126.4461386985	2.1019055715	+0.0709548658	8.850982183

TABLE II. (Continued)

s	$x_1^{(s)}$	$\log x_1^{(s)}$	$J_0[x_1^{(s)}]$	$\log \pm J_0[x_1^{(s)}]$
41	129.5878032451	2.1125641276	−0.0700895302	8.845653149
42	132.7294643885	2.1229673418	+0.0692551013	8.840451769
43	135.8711223648	2.1331271628	−0.0684497820	8.835372069
44	139.0127773887	2.1430547201	+0.0676719183	8.830408488
45	142.1544296559	2.1527603970	−0.0669199848	8.825555833
46	145.2960793452	2.1622538953	+0.0661925720	8.820809257
47	148.4377266203	2.1715442942	−0.0654883757	8.816164219
48	151.5793716314	2.1806401023	+0.0648061865	8.811616466
49	154.7210145163	2.1895493043	−0.0641448816	8.807162008
50	157.8626554019	2.1982794036	+0.0635034167	8.802797092
51	161.0042944054	2.2068374597	−0.0628808191	8.798518190
52	164.1459316346	2.2152301230	+0.0622761818	8.794321978
53	167.2875671898	2.2234636653	−0.0616886575	8.790205319
54	170.4292011631	2.2315440083	+0.0611174540	8.786165254
55	173.5708336411	2.2394767493	−0.0605618292	8.782198984
56	176.7124647027	2.2472671842	+0.0600210877	8.778303862
57	179.8540944227	2.2549203290	−0.0594945768	8.774477380
58	182.9957228701	2.2624409390	+0.0589816830	8.770717160
59	186.1373501093	2.2698335268	−0.0584818292	8.767020948
60	189.2789762004	2.2771023782	+0.0579944721	8.763386600
61	192.4206011997	2.2842515672	−0.0575190996	8.759812079
62	195.5622251597	2.2912849701	+0.0570552283	8.756295447
63	198.7038481298	2.2982062777	−0.0566024018	8.752834860
64	201.8454701561	2.3050190072	+0.0561601888	8.749428559
65	204.9870912823	2.3117265128	−0.0557281809	8.746074867
66	208.1287115488	2.3183319956	+0.0553059917	8.742772184
67	211.2703309942	2.3248385127	−0.0548932546	8.739518981
68	214.4119496545	2.3312489858	+0.0544896223	8.736313798
69	217.5535675637	2.3375662093	−0.0540947647	8.733155236
70	220.6951847537	2.3437928575	+0.0537083686	8.730041961
71	223.8368012552	2.3499314908	−0.0533301360	8.726972691
72	226.9784170965	2.3559845630	+0.0529597833	8.723946199
73	230.1200323046	2.3619544264	−0.0525970408	8.720961311
74	233.2616469051	2.3678433375	+0.0522416513	8.718016896
75	236.4032609225	2.3736534628	−0.0518933698	8.715111873
76	239.5448743794	2.3793868825	+0.0515519623	8.712245201
77	242.6864872978	2.3850455956	−0.0512172058	8.709415881
78	245.8280996983	2.3906315240	+0.0508888870	8.706622952
79	248.9697116003	2.3961465162	−0.0505668022	8.703865490
80	252.1113230227	2.4015923515	+0.0502507566	8.701142605

TABLE II. (Continued)

s	$x_1^{(s)}$	$\log x_1^{(s)}$	$J_0[x_1^{(s)}]$	$\log \pm J_0[x_1^{(s)}]$
81	255.2529339830	2.4069707427	−0.0499405637	8.698453440
82	258.3945444983	2.4122833400	+0.0496360453	8.695797172
83	261.5361545844	2.4175317339	−0.0493370302	8.693173004
84	264.6777642566	2.4227174575	+0.0490433548	8.690580170
85	267.8193735296	2.4278419898	−0.0487548620	8.688017931
86	270.9609824173	2.4329067582	+0.0484714011	8.685485573
87	274.1025909327	2.4379131406	−0.0481928275	8.682982407
88	277.2441990888	2.4428624679	+0.0479190024	8.680507768
89	280.3858068975	2.4477560258	−0.0476497924	8.678061013
90	283.5274143702	2.4525950573	+0.0473850692	8.675641520
91	286.6690215183	2.4573807640	−0.0471247098	8.673248689
92	289.8106283521	2.4621143085	+0.0468685953	8.670881938
93	292.9522348815	2.4667968154	−0.0466166118	8.668540705
94	296.0938411167	2.4714293739	+0.0463686494	8.666224446
95	299.2354470668	2.4760130382	−0.0461246021	8.663932633
96	302.3770527404	2.4805488296	+0.0458843682	8.661664756
97	305.5186581464	2.4850377379	−0.0456478493	8.659420320
98	308.6602632927	2.4894807224	+0.0454149506	8.657198846
99	311.8018681874	2.4938787130	−0.0451855807	8.654999867
100	314.9434728378	2.4982326121	+0.0449596513	8.652822935
101	318.0850772513	2.5025432949	−0.0447370774	8.650667609
102	321.2266814346	2.5068116110	+0.0445177767	8.648533467
103	324.3682853946	2.5110383851	−0.0443016697	8.646420095
104	327.5098891378	2.5152244180	+0.0440886797	8.644327093
105	330.6514926702	2.5193704875	−0.0438787324	8.642254073
106	333.7930959978	2.5234773496	+0.0436717561	8.640200656
107	336.9346991265	2.5275457389	−0.0434676814	8.638166475
108	340.0763020616	2.5315763695	+0.0432664410	8.636151173
109	343.2179048084	2.5355699358	−0.0430679701	8.634154402
110	346.3595073722	2.5395271133	+0.0428722055	8.632175826
111	349.5011097577	2.5434485591	−0.0426790865	8.630215115
112	352.6427119699	2.5473349127	+0.0424885539	8.628271950
113	355.7843140132	2.5511867967	−0.0423005506	8.626346020
114	358.9259158923	2.5550048172	+0.0421150209	8.624437021
115	362.0675176112	2.5587895644	−0.0419319113	8.622544658
116	365.2091191742	2.5625416132	+0.0417511694	8.620668644
117	368.3507205852	2.5662615238	−0.0415727448	8.618808700
118	371.4923218481	2.5699498420	+0.0413965883	8.616964550
119	374.6339229664	2.5736071000	−0.0412226523	8.615135931
120	377.7755239442	2.5772338166	+0.0410508905	8.613322583