Chapter XVII, on the calculus of variations and Hamilton's principle, carries the first subject only as far as the derivation and solution of Euler's equation, i.e., the problem of finding extremals, with applications. Most of these applications are familiar enough, though the derivation of Laplace's equation in curvilinear coordinates is not so well known. Isoperimetric problems and variable end points are considered, also integrals in parametric form. The author's insistence on accurately stated definitions of variations is to be commended. Hamilton's Principle and the Principle of Least Action occupy the last fourteen pages. Their application to a number of relatively simple problems is a most valuable feature.

Chapter XVIII, on thermodynamics and entropy, will help clear up some fundamental notions in this field.

With Chapters XIX and XX we return to matters of a more formal sort. The first takes up definite integrals with parameters, improper integrals and tests for their convergence or divergence, the Gamma and Beta Functions, improper double integrals, and special methods for the evaluation of improper integrals. The last chapter gives a brief introduction to the elementary theory of functions of a complex variable.

There are some misprints, though not an undue number. As part or all may have been corrected in the plates since the first printing, a list of them would be useless here.

D. R. CURTISS

HEATH'S EUCLID

The Thirteen Books of Euclid's Elements translated from the text of Heiberg with introduction and commentary. By Sir Thomas L. Heath, K.C.B., K.C.V.O., F.R.S. Cambridge, University Press, 1926. 8 vo. 3 volumes, pp. xii+432; 436; iv+546. £3, 10 s.

It is about eighteen years ago that this writer published in this periodical (this BULLETIN, (2), vol. 15, pp. 386-391) a review of the first edition of this noteworthy example of English scholarship and power of exposition. The titlepage then gave the author as T. L. Heath, C. B., Sc. D.; it now records honors which this and other works upon the history of mathematics have brought to the author, and with the approval of the whole scientific world. As regards the general treatment of the subject, the significance of such a publication, and its influence upon modern education, little need be added to the review above mentioned. The only matter demanding special consideration at this time relates to the changes which characterize the new edition.

In general, the work is a reprint of the first impression, with such corrections and minor changes as are naturally desirable after such a lapse of years. It is a gratifying tribute to the scholarship of the author that the changes in the original text are so few, as also that the demand for the work has made this edition necessary. It should not be thought, however, that the text has not been thoroughly revised or that it fails to include the latest information relating to discoveries in the field considered. The care shown in the revision is seen in numerous changes in the foot-notes as well as in the pages themselves. A considerable number of these changes were made for the purpose of embodying in the text certain addenda and corrigenda given in volume III of the first edition, but there are numerous other additions which were rendered necessary by recent studies of Greek literature.

Some idea of the nature of the changes may be had from the following brief summary of the more important ones:

Vol. I, pp. 8, 9. The statement regarding Euclid's *Book on the division of figures*, necessitated by Professor Archibald's edition (Cambridge, 1915) of this work.

Pp. 20–22. A reconsideration of the probable date of Heron. The author rightly speaks of it as "still a vexed question," but concludes with the statement that "the net result, then, of the most recent research is to place Heron in the third century A. D., and perhaps a little earlier than Pappus," a conclusion which he had already expressed in his *History of Greek Mathematics*. This result of his summation of the arguments will probably be accepted as conclusive unless and until new evidence is found to invalidate it.

P. 32. The question of whether or not Proclus wrote any commentaries upon Euclid's *Elements* beyond Book I,—already considered in the addenda of volume III of the first edition.

P. 39. On the origin of the name Geminus.

P. 74. On the origin of the scholia of Euclid.

P. 113. Certain recent editions of Euclid.

P. 351. Further notes on Euclid I, 47, largely from the addenda of the first edition.

P. 352. The seeming approval of Professor Peet's assertion that nothing in Egyptian mathematics "suggests" that the Egyptians were acquainted with the fact that the 3–4–5 triangle was right-angled,—an assertion which will certainly be considered further by future historians, and which (as to the word "suggested") is even now open to doubt.

P. 362. The possibility that the Pythagorean relation $a^2+b^2=c^2$ had its origin in India or in China.

The most extended additions to the text of volume I consist of two excursus. The first, which was for the most part given among the addenda of volume III of the first edition, relates to Pythagoras and the Pythagoreans and considers chiefly the arguments advanced by Junge (1907) and Vogt (1908) to the effect that the Greek work on irrationals was due rather to the later Pythagoreans than to the early founders of the brotherhood, a theory which the author believes to be untenable. He agrees, however, with Vogt that it is not likely that the early Pythagoreans actually constructed the five regular solids in the manner seen in Euclid XIII.

The second excursus will have more popular interest, since it is concerned with the history of certain fanciful names that have attached themselves to some of Euclid's theorems. These are the *pons asinorum* (properly, Euclid I, 5); "the theorem of the bride," "the bride's chair," the *dulcarnon*, or the *Francisci tunica* ("the Franciscan's cowl;" Euclid I, 47), and the *pes anseris* ("goose's foot") or *cauda pavonis* ("peacock's tail; " Euclid III, 7 and 8).

The important changes in volume II are largely from the addenda of the first edition and are found on pages 190 (on Euclid VI, def. 5) and 425 (on perfect numbers).

Volume III is practically unchanged.

The preface to this edition refers to two distinct movements "in the domain of geometrical teaching." The first relates to a "widespread desire among teachers for the establishment of an agreed sequence to be generally adopted in teaching the subject," and the second to "the movement in favor of reviving, in modified form, the proposal made by Wallis in 1663 to replace Euclid's Parallel-Postulate by a Postulate of Similarity." Such movements may be commanding the attention of teachers in England; but in the United States the movement in the first case is decidedly in the opposite direction, and the second case has attracted no attention on the part of our schools. Furthermore, it is very doubtful if a study of current textbooks would show that either of these tendencies is very pronounced in the continental countries.

In any case, however, no teacher of geometry can afford to be ignorant of the spirit of Euclid, since it is this spirit which constitutes the essence of all demonstrative geometry. It is probable that no equal expenditure of money would produce as good results in the teaching of geometry as the purchase of sets of these volumes and the placing of them in the libraries of our high schools. In comparison with the rather ephemeral educational literature of the time, a work of this kind has a permanent value and sets a scholarly standard that will encourage better teaching in the one branch of study in our secondary school that gives real insight into the significance of mathematics.

DAVID EUGENE SMITH