

Suppose that after the transformation of §1, the differential system satisfies the hypotheses stated at the beginning of this paragraph. The equations $[\phi_i]_{t=0}=0$ imply the equations $y_i^0=0$, ($i=1, \dots, r$), which imply that

$$y_i \equiv \phi_i(x_1, \dots, x_n) = 0, \quad 0 \leq t \leq T, \quad (i = 1, \dots, r).$$

The equations (2) are satisfied, therefore, for all values of $t \leq T$, and the name "invariant relations", applied to (2), is completely justified.

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DETERMINATION OF THE NUMBER OF SUB- GROUPS OF AN ABELIAN GROUP

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The present note aims to exhibit a simple method for finding the total number of the subgroups contained in an arbitrary abelian group G of order g . It will be shown that this problem can be reduced to a determination of the number of ways in which the first l independent generators of highest order of a given prime power abelian group can be selected. Hence we shall first consider this question. Therefore we shall assume for the present that g is of the form p^m , p being a prime number, and that G has k largest invariants which are separately equal to p^α . It is well known that G contains a fundamental characteristic subgroup of order p^k and of type $(1, 1, 1, \dots)$.* Each operator of order p contained in this subgroup is the $p^{\alpha-1}$ power of p^{m-k} operators of G , and no other operator of order p contained in G has this property. After selecting $l \leq k$ independent generators of highest order in a set of such generators the next independent generator of this order can therefore be selected in

$$(p^k - p^l)p^{m-k}$$

* G. A. Miller, American Journal of Mathematics, vol. 27 (1905), p. 15.

different ways, since every operator of order p^α which does not generate an operator of order p contained in the group generated by the given l independent generators can be used as such an additional generator. Hence the first l independent generators of order p^α can be selected in

$$p^{m-k} \prod_{x=0}^{l-1} (p^k - p^x)$$

different ways.

From this formula it results directly that after l_1 independent generators of order p^α in a set of such generators of G have been selected, l_2 additional such generators can be selected in

$$p^{m-k} \prod_{x=l_1}^{l_1+l_2-1} (p^k - p^x)$$

different ways whenever $l_1+l_2 \leq k$. To select a set of independent generators of a subgroup H of G , when g is of the form p^m , we may obviously proceed as follows: Consider the characteristic subgroup of G which is generated by its operators whose order is equal to the largest invariant of H . The number k_1 of independent generators of highest order in a set of reduced independent generators of this characteristic subgroup of order p^m , is evidently equal to the number of the invariants of G which are not less than the largest invariant of H . The number of different ways in which the l' largest independent generators of H in a set of such reduced independent generators can be selected from the operators of G is given by the formula noted at the close of the preceding paragraph, when m , k , and l are replaced by m_1 , k_1 , and l' respectively. After these l' independent generators have been selected we may consider the characteristic subgroup generated by all the operators of G whose order is equal to the second largest invariant of H , in case H has different invariants. The number of different ways in which the l'_2 next to the largest independent generators in a set of reduced generators of H can be selected from the

operators of this second characteristic subgroup is given by the formula of the present paragraph, since these l_2' generators have been preceded by the l' generators of highest order, in a set of reduced independent generators of H .

When the invariants of H have more than two different values we obviously repeat this process by considering next the characteristic subgroup of G which is generated by all its operators whose order is equal to the third largest invariant of H and letting l_2'' be the number of operators of this order in a set of reduced independent generators of H while l_1'' represents the number of all the larger operators in such a set. Hence it is clear that the given formulas suffice to determine the number of ways in which a set of reduced independent generators of H can be selected from the operators of G , and if this number is divided by the number of ways in which such a set can be selected from the operators of H itself, which is given by the same formulas, the quotient is equal to the number of the different subgroups contained in G which are separately of the same type as H is. This, therefore, solves the problem of determining the number of the subgroups of G whenever g is a power of a prime number. When g does not have this property G is the direct product of its Sylow subgroups and the subgroups of G are the direct products of their Sylow constituents whenever they are not prime power subgroups. The number of such subgroups having given invariants is therefore equal to the product of the numbers of the possible ways in which the Sylow subgroups involved therein can be selected from the corresponding subgroups of G . Hence the problem of determining the number of the subgroups involved in G is completely solved by the formulas noted above.