Nichteuklische Geometrie. By Dr. Max Simon. Edited by Dr. Kuno Fladt.
    Leipzig, Teubner, 1925. xviii+116 pp. 16 mo.

A careful writer always puts more pains into writing his preface, than
any other part of his work, largely in the hope of presenting his wares attrac-
tively, and inducing the reader to go further. And in most cases it is lost
labor, for the reader skips said preface. It would be a great mistake to do so
in the case of the book before us, for Fladt's preface explains the nature of
all that follows. The reader must understand, then, that this is largely a
book of piety. The story is as follows: Simon, whose knowledge of elemen-
tary geometry was unparalleled, as all students of his report on the progress
of that subject in the nineteenth century know, gave a course of lectures
in 1905, and again in 1911 on non-euclidean geometry. He probably
hoped to publish them in book form some day, but did not live to see
this wish realized, and after his death in 1918 the material was handed
to Fladt. The latter determined to publish it out of respect to his late
teacher, and to stick as closely as possible to the spirit of the original.
Simon, in a similar spirit of reverence always tried to follow as closely as
possible the classical methods of the early writers, Gauss, Bolyai, and
Lobachevsky. Hence the work was an act of piety on the part of both. But
each had to modify his original plan. Simon could not leave the subject
exactly where it had stopped a hundred years before, and Fladt, educated
in the more critical modern school, felt constrained to introduce all of
Hilbert's axioms, in order to have a really solid foundation. He shows
unusual skill in reconciling what he takes from Simon, with what comes
from Hilbert and Liebmann, but looked upon as a scientific work, rather
than as a memorial, the book has many shortcomings.

To begin with, it is almost entirely given to hyperbolic geometry, to
the exclusion of elliptic or spherical, the two being insufficiently distin-
guished any way. Then it is almost entirely "in plano"; the amount of solid
geometry is negligible. This one-sidedness is painfully characteristic.
The authors have apparently heard of none of the recent texts, which are
not in German, not even of Somerville's compendious bibliography of
non-euclidean geometry, or the earlier and less complete one of Halsted.
The endeavor is to stick closely to the euclidean model, paying as little
attention to the axioms as will "get by" in this critical age. The first five
sections are devoted to preliminary discussions, and to what one might
call "gossip" about parallels. Hilbert's axioms are introduced in §6 and
a number of theorems having no reference to parallels are deduced. Then
the axioms about parallels which distinguish euclidean, hyperbolic,
elliptic, and spherical geometry are stated and compared; the possibility of
a geometry where two lines intersect more than twice is ruled out in a foot
note. Then much care is given to showing that if one of the parallel assump-
tions is true once, it is always true. A neater way of stating this is to say
that if there exist one quadrilateral where three angles are right angles,
then if in a single instance the fourth angle is less than, equal to, or greater
than a right angle, just that will always be the case. The authors assume
the Archimedean axiom (continuity) "So weit wie notwendig." From

this point on they devote themselves almost exclusively to hyperbolic plane geometry in general, and Lobachevsky parallels in particular. It is needful to show a number of things about parallels, notably that if l is one of the parallels to m, through a point P, it is one of the parallels through every other point on itself. Much attention is given to the three types of pencils of lines, those through a point, those all parallel to one another, and those all perpendicular to a given line. The main object is to develop the formula for the parallel angle, and so the method for drawing parallels. The leading idea is taken from Engel, and consists in establishing a correspondence between a right triangle, and a quadrilateral with three right angles.

The hyperbolic circle first appears on page 62 and is defined as the locus of the reflection of a point in the lines of a pencil. When the pencil consists in concurrent lines this gives the proper circle, when it is a parallel pencil, this is the queer circle called a horocycle, when the pencil is made up of lines perpendicular to a given line this should give the equidistant curve, i.e. the locus of all points at the same distance as the given point from the given line. It will, however, only give one half of that locus. Next comes an elaborate study of the concept of area, and of trigonometry. Twice we find the functional equation

$$f(x) + f(y) = f(x+y).$$

We are told that it is "geometrically evident" that this function is continuous, hence the solution is $f(x) = rx$. But if we were content with those things which are "geometrically evident" we should not bother with non-euclidean geometry at all, for it is sufficiently evident geometrically that if Euclid's parallel axiom is not true, the amount of its falsity is not enough to disturb the equanimity of sensible people. It is not thus that real mathematics is made. The subject of three-dimensional geometry is covered in eight pages at the end. Of such interesting subjects as Clifford parallels, not a trace.

Such is the book. Interesting and clear, and impregnated with the spirit of the founders, but, at best, one of many, presenting no striking advances over others already available.

<div style="text-align: right">J. L. COOLIDGE</div>

The Principles of Thermodynamics. By George Birtwistle, Fellow of Pembroke College, Cambridge. Cambridge University Press, 1925. 163 pp.

The book is a careful presentation of the subject to students who are presupposed to have only a general knowledge of the physical science. For this reason the relation of statistical mechanics to thermodynamics, and the connection between magnetism and temperature, for which a special knowledge of dynamical or physical theory is required, are not included. The third law of thermodynamics is likewise not considered. The work is prefaced in its first chapter by concise historical and descriptive statements relating to the theory of heat, temperature measurement, laws of Boyle, Charles and Avogadro, and the conception of a perfect gas. The