

THE FORTY-EIGHTH REGULAR MEETING OF THE SAN FRANCISCO SECTION

The forty-eighth regular meeting of the San Francisco Section of the Society was held at Stanford University on Saturday, April 3, 1926. The Chairman, Professor Blichfeldt, presided. The total attendance was twenty-nine, including the following twenty-three members of the Society:

Barter, Bernstein, Blichfeldt, Buck, Cajori, Corbin, Daus, Fenner, R. L. Green, E. R. Hedrick, Hoskins, Hotelling, Sophia Levy, Libby, E. K. W. McDonald, J. H. McDonald, W. A. Manning, Moreno, F. R. Morris, Noble, Pehrson, Stager, Wong.

The Secretary read letters from Professors Bell and Carpenter of the University of Washington extending a cordial invitation to the members to be present at the regular meeting of the Section in Seattle on June 12. It was decided to hold the next Spring meeting at Stanford University, April 2, 1927.

Titles and abstracts of papers read at the meeting follow. The papers of Professor Carpenter and Mr. Cramlet were read by title.

1. Professor Florian Cajori: *Early "proofs" of the impossibility of a fourth dimension of space.*

The author cites early proofs of the impossibility of four-space given by Ptolemy, Clavius, Leibniz, Kant, Mellin and Whewell. Of these the Clavius-Mellin proof was the only one elaborated by the authors in detail; it was based upon theorems in Euclid's *Elements*, but one of those theorems was given a generality not contemplated by Euclid.

2. Professor A. F. Carpenter: *Triads of ruled surfaces.*

This paper sets up the defining system of differential equations for the projective differential geometry of a configuration of three ruled surfaces whose elements correspond in sets of three, one from each surface. A number of invariants and covariants are obtained and their geometric interpretations given.

3. Mr. C. M. Cramlet: *A determination of all invariant tensors.*

The tensor δ_ν^μ having components 0 or 1 according as $\mu \neq \nu$ or $\mu = \nu$ has the same components in all coordinate systems so may be properly regarded as an invariant tensor. A new tensor is defined by setting

$$\gamma_{s_1 \dots s_k}^{r_1 \dots r_k} = \Sigma(A_P) \delta_{s_1}^{r_1} \dots \delta_{s_k}^{r_k}$$

where the symbol Σ indicates that all products formed by permuting the indices $r_1 \dots r_k$ and multiplying these terms by $n!$ arbitrary scalars or invariants are summed. It is shown that this includes all tensors which have equal corresponding components in all coordinate systems. By choosing the scalars (A_P) as ± 1 according as they are prefixed to a product of δ 's formed by an even or odd substitution on the set $r_1 \dots r_k$, $\gamma_{s_1 \dots s_k}^{r_1 \dots r_k}$ reduces to the determinant tensor (Murnaghan, this BULLETIN, vol. 30, p. 323, and an article by the writer to appear in the ANNALS). If all of the A 's are equal to unity it reduces to a tensor here called the permanent tensor since it is useful in the study of permanents as defined by Cauchy. Both of these latter tensors will find application in the study of determinant-permanents.

4. Professor E. R. Hedrick: *On the oscillation of an arc of a curve.*

In this paper the author considers the concept of oscillation of an arc of a curve defined by Fréchet (JOURNAL DE MATHÉMATIQUES, (9), vol. 4, p. 282). The properties of this oscillation for the case of a discontinuous curve are studied, and a few properties not given by Fréchet are stated.

5. Professor J. H. McDonald: *A special involution of the fifth degree.*

It is the object of the communication to determine the involution of the fifth degree whose discriminant has three double roots. This involution reduces two hyperelliptic integrals of genus 2 and is the only case of such a twofold reduction from genus 2. Associated with this involution is another involution of order 10 which reduces two hyperelliptic integrals of genus 4. If the involution of order 5 is represented on a conic the involution curve is a rational quartic which has the property that the lines joining each node to the points of contact of a corresponding double tangent form with the tangents at the node an harmonic pencil. This quartic depends on two arbitrary parameters.

6. Professor J. H. McDonald: *The relation between Goursat's cubics.*

In the reduction of hyperelliptic integrals by a transformation of order 3, the two cubic factors of the reducible sextic have a simple relation to each other when taken in Goursat's normal form. The invariantive form of this relation is obtained and is of degree 8 in the coefficients of each cubic.

7. Professor J. H. McDonald: *On unicursal curves.*

If the coordinates of a point on a curve are rational functions of degree n of a parameter t the curve is in general of degree n , but the degree may be a submultiple n' of n , and the coordinates may be expressed as rational functions of degree n' of a parameter t' which is a rational function of t of degree n/n' . A means of determining when such a reduction takes place is found and also the function t' of t . Necessary and sufficient conditions that the curve should be of degree n were found in a different form by Humbert. A determination of the function t' of t was not given.

8. Dr. B. C. Wong: *On the correspondence between space sextic curves and plane quartics in four-space.*

This paper will appear in full in this BULLETIN.

9. Dr. J. D. Barter: *Note on skew symmetric determinants of higher dimensions.* Preliminary report.

The object of this investigation is to determine to what extent the important theorems on skew-symmetric determinants of two dimensions, connected with the names of Cayley and Frobenius, may be extended to higher dimensions. The theory is treated as a chapter in that of the invariants of n -ary p -vectors. The properties of Pfaffians are examined and it is found that generalization is rather restricted.

10. Dr. J. D. Barter: *Note on commutative matrices, roots of matrices and induced transformations.*

The theory connected with these subjects is treated in those directions in which the existing results appear incomplete or the methods unnecessarily cumbrous. By elementary methods associated with the notion of polar series the expressions for matrices commutative with a given matrix, and of any root of this matrix are obtained. The leading properties of the induced transformation are exhibited.

B. A. BERNSTEIN,
Secretary of the Section.