Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik. By Adolph Kneser. 2d edition. Braunschweig, Vieweg, 1922. 8+292 pp.

Kneser's *Integralgleichungen*, which appeared first in 1911, was a pioneer in its field, and during the time which has elapsed since then, it has remained the only thing of its kind. To be sure, the book of Heywood and Fréchet emphasizes the physicial applications, and the problems of potential theory, in particular, have been methodically developed by means of integral equations. But the present work is, as far as the reviewer knows, unique in its inclusion of and emphasis on the application of the Hilbert-Schmidt theory to important one-dimensional problems.

The first edition was ably reviewed in the BULLETIN* by W. A. Hurwitz. The changes which have been made consist in about eighty pages of new or rewritten material, the remaining pages of the text having been reprinted without other change than an occasional alteration in notation. The most significant feature of the revision is the assembling and rounding out in one chapter (III) of a general theory of the symmetric kernel. The presentation follows the lines of the Schmidt theory, but it is further enhanced by the addition of paragraphs on Mercer's theorem on the convergence of the development of a continuous definite kernel in terms of its characteristic functions, and on Weyl's theorem on the diminishing effect on the positive characteristic numbers of a kernel caused by adding to the kernel a positive definite kernel.

The author finds a physical problem leading to an unsymmetric kernel in the vibrations of an elastic bar, account being taken of thermal effects. For this problem, the development question is treated by means of the theory of residues. Difficulties due to multiple characteristic numbers are not here encountered. There follow indications for a similar treatment of Sturm-Liouville problems in which the coefficients are not always necessarily real. The case in which the fundamental interval extends to infinity is then considered, and Hilb's generalization of the Fourier integral is derived and illustrated in the known cases of the Fourier integral and the corresponding integral in Bessel functions.

Another new topic arises from the treatment of a boundary value problem connected with electric cables, in which a boundary condition depends on the parameter. Here, ordinary orthogonality is replaced by a

* Vol. 19 (May, 1913), pp. 406–11. For other analyses and reviews, see *Lacour*, BULLETIN DES SCIENCES MATHÉMATIQUES, vol. 46 (1911), pp. 254–61; Korn, ARCHIV FÜR MATHEMATIK UND PHYSIK, (3), vol. 18 (1911), pp. 82–83; Plancherel, L'ENSEIGNEMENT MATHÉMATIQUE, vol. 13 (1911), pp. 428–29.

"weighted" (belastete) orthogonality: $V_1(1)\,V_2(1)+\int_0^1 V_1(x)\,V_2(x)\,dx=0$. Theorems analogous to the usual ones are indicated.

The bibliography in the appendix should not be overlooked. While it lays no claim to completeness, it is particularly valuable in the field of the applications. Its extent has been nearly doubled in the new edition.

The additions have materially enhanced the value of the book, and the chapter on the theory of the symmetric kernel has helped to meet Hurwitz' criticism as to the confusing effect of frequent alternation of general theory and particular examples. But only partially so, for as a rule several pages must be read before one can ascertain the precise conditions for the validity of a theorem, and in some cases the reader must bring an independent judgement to bear on his quest (e. g. on p. 38, line 19; the functions must be continuous and have *continuous* derivatives of first and second orders as well as piecewise continuous derivatives of third and fourth orders if the reasoning indicated is to establish the stated results). The style is further complicated by the habit of deferring the statement of a theorem until after its proof. Thus is imposed upon the attention of the reader the double task of following the reasoning and endeavoring to determine its import. Nor is much help given him by preliminary elucidation as to the general goal or the salient features of the discussion to follow.

The lacuna pointed out by Hurwitz in the proof of the theorem that to a solution of a homogeneous integral equation there always corresponds a solution of the associated equation has been allowed to stand; the section has been reprinted with such fidelity that a confusing typographical error recurs (p. 249, line 16: "Gleichung (7)" should read "Gleichung (3)").

The preparation requisite for a profitable reading of this book includes a knowledge of the rudiments of integral equations, some acquaintance with differential equations, with the theory of functions, and with physics; above all some mathematical maturity is essential. For one so equipped it is highly interesting and suggestive. Certainly no one who has to lecture on integral equations can afford to be unacquainted with its contents.

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Introduction à la Théorie de la Relativité, Calcul Différentiel Absolu, et Géométrie. By H. Galbrun. Paris, Gauthier-Villars et Cie., 1923. x+457 pp.

This work is a rather complete treatment of the mathematics of the relativity theory. Three chapters, about 100 pages, are devoted to a systematic and detailed exposition of the method of the absolute differential calculus. The differential geometry of n-dimensional space is allotted four chapters including slightly over 150 pages. The remainder of the book is devoted to mechanical and electromagnetic