

Substance and Function, and Einstein's Theory of Relativity. By Ernst Cassirer (Translated by W. C. and M. C. Swabey). Chicago, Open Court, 1923. xii + 465 pp.

Such mathematicians as have sympathy with the philosophy of Immanuel Kant will find in the first part of this volume a valuable discussion of the foundations of mathematics from their special point of view.

Turning to the author's treatment of the philosophical aspects of the theory of relativity, we find (p. 430) the following remarkable statement: "Physics now proves not only the possibility, but the reality of non-Euclidean geometry; it shows that we can only understand and represent theoretically the relations which hold in 'real' space by reproducing them in the language of a four-dimensional non-euclidean manifold." Since, however, any non-euclidean m -space may always be embedded in a Euclidean space of not more than $\frac{1}{2}m(m+1)$ dimensions, it follows that the question whether the physical world is euclidean is meaningless.

Furthermore, if we admit discontinuous mappings, any continuum of m dimensions may always be mapped upon a continuum of n dimensions ($m \neq n$). Therefore the question "Is the physical world four-dimensional?" is quite meaningless. In the sentence quoted above we have a good example of the fallacy involved when a philosopher takes a science, such as physics, at a given stage of development and concludes that its methods constitute final evidence as to the alleged restraints which the structure of the mind of man imposes upon so much of the world as may be intelligible.

Mathematicians create tools of considerable variety which the physicist may use whenever they suit his purpose. If, for example, he were to find that the consideration of the world line of a particle as the integral curve in a euclidean ten-space of an ordinary homogeneous linear differential equation of the eleventh order yielded significant results, then he would be at liberty to use that method. Accordingly, many of those who do not accept Bertrand Russell's exclusion of metaphysics and ontology from the domain of significant philosophy would doubt the existence of such metaphysical or ontological implications of the physicist's choices between mathematical tools as are drawn by Cassirer. Those who do accept Russell's restriction of philosophy to the field of logic would find Cassirer's discussion of relativity of no interest.

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