

THE SEPTEMBER MEETING OF THE
SAN FRANCISCO SECTION

The forty-second regular meeting of the San Francisco Section of the American Mathematical Society was held at the University of Southern California, Los Angeles, on September 18, 1923. Professor Cajori, chairman of the section, opened the meeting and presided until the election of the new chairman. The attendance included the following seventeen members of the Society:

Bateman, Cajori, A. F. Carpenter, Daus, Glazier, Hoskins, Glenn James, F. R. Morris, Russell, Sherwood, Showman, Steed, Touton, Wear, Willett, Winger, Wolfe.

The election of officers for the ensuing year resulted as follows: Chairman, Professor A. F. Carpenter; Secretary-Treasurer, Professor B. A. Bernstein; Program Committee, Professors B. A. Bernstein, D. N. Lehmer, E. T. Bell.

Titles and abstracts of the papers read at this meeting follow below. The papers of Professor Bell, Griffin, and Porter, and Mr. Romig were read by title. Mr. Romig was introduced by Professor Cajori.

1. Professor E. T. Bell: *Notes on recurring series of the third order.*

Important new historical material regarding the u , v of Lucas is given in the first part of this paper. The remainder is devoted to the algebra of the three linearly independent functions defined by a recurrence relation of the third order. The most general possible addition theorems for each of the functions are obtained. By slight changes in notation all results of the paper can be extended at once to series of order $n > 3$. As was to be expected, the results are much more complicated than those for Lucas' functions. There are a few applications to Diophantine analysis.

2. Professor E. T. Bell: *Analogies between elliptic functions and the u , v of Lucas.*

This paper appeared in full in the November, 1923, number of this BULLETIN.

3. Professor Florian Cajori: *Did Pitiscus use the decimal point?*

While Pitiscus in his *Trigonometry* of 1612 used fractions extensively, he did not use the point as separatrix between units and tenths. Most modern historians of mathematics state incorrectly that he used the decimal point in his trigonometric tables of 1612. The point does occur hundreds of times, but simply as a mark to divide the numeral figures into groups easier to read and copy.

4. Professor Florian Cajori: *Empirical generalizations on the growth of mathematical notations.*

Several empirical observations on the nature and growth of notation are given, which point to haphazard developments usually failing of general adoption, and which set forth the difficulties encountered in securing universal notations. The efforts of international committees towards uniformity of notation are recounted.

5. Professor A. F. Carpenter: *Linear complexes associated with the general ruled surface.*

In a paper read before the San Francisco Section of the Society in April, 1923, a number of theorems were proved concerning the primary and secondary flecnodal and complex cubics associated with each element of a ruled surface. Each of these cubics determines a linear complex. In the present paper, the author discusses the properties of these complexes and certain new cubics defined by means of them.

6. Professor A. F. Carpenter: *Note on a net of curves on a general surface.*

By a method analogous to that which enables us to define lines of curvature of a surface, and the Dupin indicatrix at a point, it is possible to obtain a quadratic differential form whose vanishing determines the extreme values of the geodesic torsion, and which may therefore be taken as defining a net of curves on the surface. These curves, which may properly be called lines of torsion, constitute an orthogonal system, the two curves of this system through any surface point bisecting the angles between the lines of curvature. The maximum and minimum values of the geodesic torsion are equal but opposite in sign for all

real surfaces, and the indicatrix for the geodesic torsion is a rectangular hyperbola whose asymptotes coincide with the tangents to the lines of curvature.

7. Dr. P. H. Daus: *Numerical illustrations of a theorem in ternary continued fractions.*

Connected with the equation $x^3 + qx - (p_1^3 + p_1q + 1) = 0$, there exists a normal ternary continued fraction expansion $(p_1, p_1^2 + q; 2p_1, 3p_1^2 + q; 3p_1, 3p_1^2 + q)$. This note considers certain numerical cases in the light of a theorem by Lehmer (PROCEEDINGS OF THE NATIONAL ACADEMY, vol. 4 (1918), p. 360) which determines whether the ratios $A_n : B_n : C_n$ formed from the n th convergent set do or do not approach limits.

8. Mr. H. P. Robertson: *The absolute differential calculus of a non-Pythagorean, non-Riemannian space.*

Fundamental in the theory of relativity is the assumption that the line element of four-dimensional space-time is represented by the square root of an invariant quadratic differential form, and the subsequent generation of its related tensors by means of the absolute differential calculus. If, instead, it is assumed (a possibility suggested by Riemann) that the line element is represented by the $2m$ th root of a differential form of degree $2m$ (m an integer) a new calculus is needed for the generation of tensors connected therewith. The present paper develops such an instrument, which applies to differential forms of degree $2m$ in n variables, and in which the Ricci-Levi-Civita calculus is included as a special case.

9. Dr. Victor Steed: *On a system of equations connected with the lines on the cubic surface.*

To show that there are lines on the cubic surface, Salmon suggested eliminating two of the cartesian coordinates from the equations of a line and the equation of the surface, and then equating to zero the coefficients of the resulting equation in the third coordinate. The four equations in the four coordinates of the line thus obtained are here interpreted as four cubic hypersurfaces in space of four dimensions. If these were not special, they would have, by the theorem of Bezout, 81 distinct points of intersection. The

question of how to account for the fact that these hypersurfaces have only 27 points of intersection in finite space is considered, it being shown that all four hypersurfaces have in common three skew lines lying entirely in the hyperplane at infinity.

10. Professor Harry Bateman: *Derivation of three-dimensional potentials from four-dimensional potentials.*

When a solution of Laplace's equation in four variables is integrated with respect to one of these variables from $-\infty$ to $+\infty$ the result is generally a solution of Laplace's equation in three variables. Since potential functions in four variables are often easier to calculate than analogous potential functions in three variables, the well known device just mentioned may be of real value. At any rate, it leads to some interesting identities, as is shown in this paper.

11. Professor F. L. Griffin: *Curves functionally conjugate with respect to a given curve.*

The author defines a general category of relationships among coplanar curves as follows: Let the radii vectores of three coplanar curves be q_1, q_2, q_3 , and let there exist among the radii corresponding to any common value of θ a functional relation $F(q_1, q_2, q_3) = 0$. Regarding one of the curves as a basic reference curve, the other two will be called *functionally conjugate* with respect to the first. Ordinary inverse curves, cissoids of curves, etc., constitute special classes of such conjugates. For various simple selections of the function F and the basic curve, the conjugates of conics are found to be well known curves. This type of relationship admits of generalization to any number of curves and of extension to surfaces and loci in any number of dimensions.

12. Professor M. B. Porter: *Second mean-value theorem for integrals of summable functions.*

This paper appeared in full in the November, 1923, number of this BULLETIN.

13. Mr. H. G. Romig: *Division by zero (a historical study).*

The author describes the first use of zero as a divisor among the Hindus in 628 A. D., and traces historically the

observance of the difference between the infinitesimal and absolute zero, the rigorous exclusion of division by absolute zero by Martin Ohm in 1828 and Axel Harnack in 1881, and the more recent treatment of division by absolute zero in algebra, modern geometry, and function theory.

14. Professor R. M. Winger: *Singular points of self-projective rational septicimics.*

In an earlier paper, the author obtained the types of self-projective rational septicimics. In the present paper he analyzes the singularities which occur in great variety and considerable complexity. The standard methods of expansion in the neighborhood of the singularity and birational transformation are not suitable for rational curves which are normally given in parametric form. Moreover, the double-point equation is tedious to expand since it leads to an equation of degree 30 in the case of the septicimic. Dr. Scott's theory of excess is, however, of great assistance. Two general theorems of interest are proved: (1) If a rational curve of order n is invariant under a cyclic group of order n (and no higher group) this group cannot contain an homology. (2) A rational curve of order n which is invariant under a cyclic G_n (and no higher group) has a multiple point of order greater than 2, having, however, but two distinct parameters. The parameters of the multiple point are the fixed parameters of the binary group, while the multiple point is the only fixed point of the ternary group which lies on the curve.

L. M. HOSKINS,

Acting Secretary of the Section.