Fundamental Congruence Solutions. By Lt.-Col. Allan Cunningham. London, Francis Hodgson, 1923. xviii + 92 pp.

Haupt-exponents, Residue-indices, Primitive Roots, and Standard Congruences. By Lt.-Col. Allan Cunningham. Francis Hodgson, 1922. viii + 136 pp.

The first of these two books gives in 92 pages one value of the root y, of the "fundamental congruence":

$$y^\xi \equiv + 1 \ (\text{mod } p \text{ or } p^k)$$

for values of the modulus not exceeding 10,201. ξ is the "Haupt-exponent," or the exponent to which y belongs. Owing to the immense amount of labor necessary to determine it, the smallest value of y is not always the one given. If, however, the smallest value is not greater than 13, it is given. Also when $\xi = p - 1$ (and y is therefore a primitive root), the smallest value of y is listed. There are further cases, when ξ has certain other values, where the root y is the smallest root.

One root, y, being listed, the others may be determined. Rules are given to effect this. Some account of the method of computation of the tables appears in the introduction, together with a description of the checks employed.

The tables are arranged in two lines for each modulus, thus:

$$
\begin{array}{c|c}
p & \xi, \nu \\
\hline
 & y
\end{array}
$$

where ν is the complementary factor of ξ with respect to $p - 1$, so that $\xi\nu = p - 1$. The values of the modulus are arranged consecutively so far as the primes are concerned, but for some reason, not explained, and not quite clear to the reviewer, those moduli which are powers of primes are inserted, part on the eighth page and part on the ninetieth page. These lists are not hard to find when one knows where they are, but a note telling where they are hidden would save users of the table some little annoyance and bewilderment.

In 31 pages of the second book we are given for each prime p not greater than 10,000 the factorization of $p - 1$. Then is given also the exponent to which each of the numbers 2, 3, 5, 6, 7, 10, 11, 12 belongs. A primitive root for each prime is also indicated. Then comes a similar table for the powers of 2, followed by a page giving the same information for powers of odd primes. Pages 33 to 34 give the solutions (x_0, α_0, x_0') of the two congruences

$$
\begin{aligned}
2^{x_0} &\equiv \pm y^{\alpha_0} \\
2^{x_0'} y^{\alpha_0} &\equiv \pm 1
\end{aligned}
\quad (\text{mod } p \text{ or } p^k)
$$

for $y = 3, 5, 7, 11$, and p as before. A similar table follows in which the congruences are

$$
\begin{aligned}
10^{x_0} &\equiv \pm y^{\alpha_0} \\
10^{x_0'} y^{\alpha_0} &\equiv \pm 1
\end{aligned}
\quad (\text{mod } p \text{ or } p^k).
$$

The final table of the book, beginning at page 97, is a continuation of the first, slightly abridged to accommodate the larger numbers that appear as p runs from 10,007 to 25,409 inclusive.

The value of a piece of tabular work may appear in either or both of two ways. The table may be of use in other computations; or it may serve to put complicated results before the eye in such a simple form as to reveal hidden relations. The first of these purposes is evidently the one intended for the two books of tables published by Lt.-Col. Cunningham. They are to serve as aids in the factorization of numbers of the form

$$y^n \pm 1.$$

A detailed explanation of their use in this connection, together with illustrative examples, would do much to increase their usefulness and availability to other workers in this field. The need for an extensive table of primitive roots is readily appreciated by any one working in the theory of numbers. If one should by chance encounter a congruence such as

$$y^{356} \equiv 1 \quad (\text{mod } 4297)$$

he would turn to these tables with gratitude, but just how a congruence of this sort might arise in connection with other parts of the theory of numbers is a question to which at least a few words might well be given.

D. N. LEHMER

Vector Calculus. By James Byrnie Shaw. New York, D. Van Nostrand Company, 1922. iv + 314 pp.

In his preface the author says that "he has examined the various methods that go under the name of vector, and finds that for all purposes of the physicist and for most of those of the geometer, the use of quaternions is by far the simplest in theory and in practice." This indicates clearly the point of view of the book. The quaternion notation is used but tables of other equivalent notations are given.

The first chapter is a historical sketch of the various systems of vector analysis. The next six chapters are concerned with scalar and vector fields, the algebraic combinations of vectors and the differential operations. These are illustrated by a large number of quantities occurring in geometry, electricity and magnetism, mechanics, theory of elasticity, etc., each of which is defined when introduced. The following two chapters give a systematic exposition of the differential and integral calculus of vectors, with applications to geometry and such topics as Laplace's equation, Green's theorem, and spherical harmonics. The remaining three chapters treat the linear vector function with applications to deformable bodies and hydrodynamics. Extensive lists of problems are given covering almost all the topics discussed.

The book impresses one as containing an extraordinary number of topics treated in a way that (to one acquainted with those topics) is interesting and easy to follow. Students of the better class will certainly acquire a considerable knowledge of mathematics and physics by studying this book. Whether an individual teacher chooses it, however, will probably depend on whether he is willing to use the quaternion notation or translate it into the form that he does use.

H. B. PHILLIPS