Lezioni sulla Teoria dei Numeri algebrici e Principi d'Aritmetica analitica. By Luigi Bianchi. Pisa, Enrico Spoerri, 1921. viii + 444 pp.

This work consists of a 50-page introduction and three chapters. The introduction contains a development of the theory of numbers for the field $k(i)$. The theorems of the theory of the rational numbers are extended to this field giving the proof of the unique separation into prime factors, the development of the φ function, the proof of Fermat's theorem, the theory of binomial congruences with proof of the existence of primitive roots of the congruence $x^{\varphi(p)} \equiv 1 \bmod p$, and the theory of indices and quadratic residues with the law of reciprocity.

The first and second chapters are devoted to the theory of algebraic numbers in a general field of degree n. The first chapter contains the development of that part of the theory which is independent of factorization. The existence of a base is shown and the discriminant of a field is defined. The greater part of the chapter is devoted to the theory of units which is developed from Minkowski's theorem on linear forms leading up to the proof of the existence of a fundamental system of units.

The second chapter contains the theory of ideals, showing the purpose of their introduction into the theory of algebraic numbers, proving the unique factorization theorem, and discussing congruences with respect to an ideal modulus. The chapter also contains the theory of the classification of ideals, the discussion of the group of classes, and the correspondence between the classes of ideals and equivalent decomposable forms.

The last part of the chapter is given over to a brief consideration of the theory of ideals in Galois, Abelian, cyclotomic, and Kummer fields. The law governing the factorization of the rational primes in a cyclotomic field of prime degree is developed. This is also done earlier in the chapter for quadratic fields as an application of the theory of quadratic residues with respect to an ideal modulus.

The last chapter contains a good presentation of the analytic theory of ideals. The Riemann and Dedekind zeta functions are studied showing the relation of the latter to the problem of the determination of the number of classes in a field.

The zeta functions for quadratic and cyclotomic fields are reduced to their simpler forms and the method for the determination of the number of classes in a quadratic field is given for the case when the discriminant of the field is congruent to 1 mod 4.

The last paragraph contains a brief indication of Hecke's results regarding the zeta function.

G. E. WAHLIN

Einführung in die elementare und analytische Theorie der algebraischen Zahlen und der Ideale. By Dr. Edmund Landau. Leipzig and Berlin, B. G. Teubner, 1918. viii + 143 pp.

This work is divided in two parts. The first part (54 pages) contains a brief, but unusually clear, presentation of the elementary theory of algebraic numbers and ideals in an algebraic number field.

The author states in the preface that the aim of the work is not to give an exhaustive treatment, but to bring the reader up to and a little beyond the fundamental theorem of Dedekind that the separation of an ideal into the product of prime ideals is unique.

The subjects considered beyond the work necessary for the proof of the theorem mentioned are: the theory of the classification of ideals showing that the number of classes is finite, and the theory of units in an algebraic number field, with the proof of the existence of a fundamental system of units.

The second part is devoted to the analytic theory of ideals. The first chapter gives a clear introduction to the functions $\zeta(s)$ and $\zeta(s;\ \kappa)$, where κ is a class of the field. It is shown that, except for a pole of the first order in $s = 1$, the functions are regular in the entire plane, and that the residue of $\zeta(s;\ \kappa)$ in $s = 1$ is a number λ, independent of the class κ, while that of $\zeta(s)$ is λh, where h is the number of classes. Hecke's functional equations for $\zeta(s)$ and $\zeta(s;\ \kappa)$ are also developed.

The second chapter is a study of the distribution of the zeros of $\xi(s)$, and the third chapter leads to the proof of the author's remarkable theorem that, asymptotically, the number of prime ideals is the same in all fields.

The last chapter contains the result of the author's researches regarding the number of ideals, in a field or a class, whose norms are less than x. If this number be denoted by $H(x)$ for the field and $H(x;\ \kappa)$ for a class, it is shown that

$$H(x;\ \kappa) = \lambda x + O(x^{\vartheta})$$
$$H(x) = \lambda h x + O(x^{\vartheta})$$

where λ and h have the meaning given above,

$$\vartheta = 1 - \frac{2}{n+1}$$

and $O(x^{\vartheta})$ is a function whose quotient by x^{ϑ} is limited for sufficiently large x. It is shown that the exponent ϑ cannot be less than

$$\frac{1}{2} - \frac{1}{2n}.$$

The last four pages contain a brief historical survey with examples from a quadratic field, as well as notes of reference to the literature bearing on the various sections.

G. E. WAHLIN

Lehrbuch der Funktionentheorie. By L. Bieberbach. Band I. *Elemente der Funktionentheorie.* Leipzig, Teubner, 1921. 6 + 314 pp.

Funktionentheorie. By L. Bieberbach. Teubner's Technische Leitfaden, Bd. 14. Leipzig, Teubner, 1922. 118 pp.

The author in his preface to the first book pleads guilty to entertaining the hope that he has written a text-book on complex function theory. He proceeds to set forth the qualities that such a text-book should have, viz: completeness, clarity, simplicity and unity. That the author succeeds in giving a clear, elementary presentation of the fundamental principles of the theory of functions of a complex variable cannot be gainsaid.

The table of contents indicates the usual order of topics.