SHORTER NOTICES

Notes d'Histoire des Mathématiques (*Antiquité et Moyen Âge*). By B. Lefebvre, S. J. Louvain, Societé Scientifique de Bruxelles, 1920. viii + 152 pp. Paper.

This little work is made up of a series of articles published under the title "Autour d'une Histoire des Mathématiques" in the REVUE DES QUESTIONS SCIENTIFIQUES during the years 1907–1911. These articles have been revised since their original appearance and are now published in the hope that they may be, as they will be, of further service to teachers and students.

The author is a member of an order that has produced many mathematicians of note, many devoted teachers, and a body of self-sacrificing men who, in the seventeenth century, carried Western mathematics to the Far East. Such men as Ceva, Cavalieri, Clavius, Matteo Ricci, Verbiest, and Vincenzo Riccati, to name only a few, have worthy successors, as teachers, in such Belgian scholars as Vanhée, Bosmans, and Lefebvre—men who, in the years of the holocaust, suffered great privations—Vanhée being imprisoned at hard labor, Lefebvre seeing the library at Louvain destroyed and his people killed, and Bosmans spending the years in relieving the destitute in Brussels. It is, therefore, a pleasure to see, in such a work as this, an evidence of the spirit to "carry on," and to bring Belgium back to a full appreciation of her standing as a home of scholars.

The work is divided into fourteen chapters, as follows: I. The historians of mathematics; II. Oriental mathematics; III. Mathematics of Rome, of Byzantium, and of the Arab schools; IV. Numeration of the Greek, Roman, and Medieval scholars; V. Hindu origin of our numerals; VI. The development of the Hindu-Arabic system; VII. Mathematics in the Middle Ages in general; VIII. The early Middle Ages; IX. The time of Charlemagne; X. Gerbert (Sylvester II); XI. The exact sciences in Belgium; XII. First Renaissance of mathematics in the twelfth and thirteenth centuries; XIII. The Arabs in Europe; XIV. The School of Toledo in the twelfth century. Such an array of titles is tempting to the general reader as well as to the teacher and the mathematician.

Père Lefebvre is modest in his claims. He frankly states that his work, say the equivalent two hundred pages of the more conventional size, is merely a set of notes; and yet a set of notes of this extent may be very valuable, or it may be useless, or even worse. It is needless to say, however, that the work of a man of Père Lefebvre's erudition can not be other than helpful to students of mathematics, and interesting and suggestive, to say the least, to scholars in general.

His first chapter, on the historians of mathematics, is essentially a critical review of Ball's well known work, but it contains a judicial estimate of the works of other historians, such as Montucla, Tannery, Chasles, Zeuthen, and Cantor, not to speak of such lesser and often inferior writers as Marie and Hoefer. As to Mr. Ball, the criticisms extend throughout the work, and are directed not only against the translation, which was carelessly made, but against the original text, especially for its lack of facsimiles to illustrate such matters as the ancient methods of writing equations, for its neglect of oriental mathematics, for its failure to make use of such researches as those of Tannery, and for such doubtful statements as those relating to the discovery of the conic sections by Menæchmus and the destruction of the Alexandrian library by the Christians.

The criticism that little or no attention is paid by Mr. Ball to the contributions of Boethius, Capella, Cassiodorus, and Varro is more easily answered, since the English historian had to limit his space, and on the whole he seems to have chosen fairly well. If he should wish to return Père Lefebvre's criticism in kind, he might well ask what he should have omitted in order to find space for such names as those mentioned. He might even go further and inquire by what right his critic should speak of "Mohammed-ben-Mouça Al-Hovarezmi, *surnommé* Al-Khorizmi," since "Al-Hovarezmi" and "Al-Khorizmi" are merely different transliterations of the same Arabic name,—the latter being the better of the two. Indeed, it may well be asked why the French writers have, in general, been so lacking in uniformity in their transliterations of all oriental names, a difficulty that foreign readers find very troublesome. Mr. Ball might also ask why Mohammed ibn Musa's name also appears as Al-Hovarez (p. 22), Al-Kharizmi (p. 121), and Al-Khowarez (p. 133), and whether these various forms are not unnecessarily confusing. He might also inquire as to the validity of the statement (p. 23), "Le surnom Al-Khorizmi devint le titre, *dans le langage courant des Arabes* (Italics mine), d'un ouvrage du même vieil auteur sur l'Arithmétique hindoue." This statement is, so far as this reviewer is aware, an unwarranted one, and it would be interesting to have it confirmed. The facts seem to be that the medieval Latin translators did their best to put the phrase "the book of al-Khowârizmi" into their tongue, and the best they could do was to write *Liber Algorismi*, or *Liber Algoritmi*, whence came our "algorism," "augrim," and other variants. Mr. Ball might also ask the authority for such a statement as that the Greeks (presumably, of course, those of the classical period) knew the proof of nines, and that it may have been known to the Pythagoreans. If any unquestionable authority for these statements exists, this reviewer is not aware of it.

On the other hand, Père Lefebvre has given his readers much that will be very helpful, as in his brief but scholarly treatment of the Roman, Greek, and Hindu-Arabic numerals, in his notes on the French abacus, and particularly in his statements as to the debt we owe to the Christian Church in the Dark Ages. We have no better essay upon the contributions of the medieval monks to the preservation if not the advance of mathematics in the centuries from the ninth to the thirteenth than Père Lefebvre has here set forth. This might naturally be expected, for a churchman is writing of other churchmen, and he does so not merely with a desire to place their work in the proper light, but with a knowledge, derived from an access to material, that few others possess. It is here, indeed, that the work assumes a value all its own, and one which renders it a necessity in the library of the student of the history of mathematics.

Père Lefebvre's work shows the effect of writing and publishing his essays at different times. There is a lack of that perfect coordination which comes from uninterrupted labor. Moreover, the work, as has been shown above in a few instances, has various slips of the pen or of the memory, little errors, in the main, that it is quite unnecessary to mention in a brief review. The treatment of the Middle Ages is so scholarly and helpful as far to outweigh the minor imperfections that, considering the tumult of recent years, can easily be overlooked. Rather than search for errors of no moment, we should express our indebtedness to one who, with all his cares in these troublous times, has collected his material and made it more available for historians and students of mathematics.

DAVID EUGENE SMITH.

Lezioni di Meccanica Razionale, Seconda Edizione. By Pietro Burgatti. Bologna, Nicola Zanichelli, 1919. xi + 544 pp.

The first edition of this work was published in 1916. Although the present edition contains about fifty pages more than the first, the topics treated and the method of treatment remain unchanged; the additional pages being due to brief additions scattered throughout the various chapters. There are no exercises, but it is stated in the preface that the author intends to publish a separate book of exercises with solutions.

The book begins with a chapter on vector analysis containing as much of the subject as is needed for the development of the mechanics of a rigid body, which is taken up in the immediately following chapters. This development, aside from the fact that vector methods are used, is pretty much along traditional lines, i.e., kinematics of a particle and of a rigid body, including some discussion of the geometry of motion; statics, in which problems are first solved by writing the equations of equilibrium and later by the method of virtual work; dynamics, which includes the theory of the top, generalized coordinates, and a brief chapter on the problem of three bodies. Before taking up the last two chapters which give a brief introduction to the mechanics of deformable bodies, the author finds it necessary to insert another chapter on vector analysis, in which are considered such matters as divergence and curl of a vector, the theorems of Green and Stokes, and Poisson's equation. The closing chapter deals with the historical development of mechanics.

It is recognized that the student of mathematical physics must be familiar with vector analysis, and texts on electricity and magnetism generally begin with a mathematical introduction which gives the machinery required for what is to follow. More recently the same idea is being extended to mechanics. We now have a number of books which treat the mechanics of a rigid body and also the mechanics of continua by vector methods, but the number is not so large but that the book under review can find a hearty welcome. So much of the subject of mechanics is essentially geometric that it is particularly well adapted for study by vector methods, and any careful teacher, whether he uses the formal vector notation or not, is sure to bring out the geometric interpretation. The thing that must be remembered, however, is that even when a property