

Moreover, if $\psi(x, y)$ is the most general function satisfying all of these conditions, we can put $\psi(x, y) = f(x, y) - f(y, x)$ without loss of generality. A brief discussion of such questions is given in section III of the paper cited. Hence from the identity for $f(x, y)$, we infer at once that

$$\begin{aligned} \Sigma d_3[\psi(d_1 + d_3, d_1 + d_2) + \psi(d_1 - d_3, d_1 + d_2)] \\ = \Sigma d_3[\psi(d_1 + d_3, d_1 - d_2) + \psi(d_1 - d_3, d_1 - d_2)], \end{aligned}$$

and this is the formula (Q) of Liouville.

The 39 forms of the addition theorems given by Jacobi in section 18 of the *Fundamenta Nova* imply a multitude of such consequences, many of which are of arithmetic interest.

The author wishes to express his indebtedness to Professor Frank Nelson Cole for encouragement and inspiration, not only in this paper, but for much of his other mathematical work.

UNIVERSITY OF WASHINGTON,
January 19, 1921.

SHORTER NOTICES

General Theory of Polyconic Projections. By Oscar S. Adams. Washington, United States Coast and Geodetic Survey, 1919. Special Publication No. 57. 174 pp.

There are many ways of representing, or projecting, the surface of the earth, or parts of it, upon a plane. Any system of lines may be chosen to represent the parallels of latitude, and a second system to represent the meridians. The book before us is designed to give a full account of the so-called polyconic projection, that is the projection in which parallels of latitude are represented by arcs of a non-concentric system of circles with collinear centers. The line of centers is usually, but not necessarily, taken for the central, or principal, meridian. The mathematical problem consists in setting up the equations for the meridians under various hypotheses, methods for constructing the meridians, spacing the parallels, determining the magnification, and so forth. These details the author has worked out for various cases, deriving the formulas for the ellipsoid as well as for the sphere.

Stereographic projection is one type of polyconic projection.

The author devotes considerable space to it, taking for the plane of projection the equatorial plane, or the plane of a meridian, or the plane of the horizon of a given place.

Another important case is that in which a given parallel is represented by a circle whose radius is equal to an element of the tangent cone to the earth's surface along the parallel and included between the point of contact and the vertex. Parallels are spaced along the central meridian in proportion to their true distances along this meridian. This is the case usually referred to as polyconic projection. (See the article on *Mathematical Geography* by Col. Sir A. R. Clarke in the *ENCYCLOPAEDIA BRITANNICA*.) This projection has been used extensively by the U. S. Coast and Geodetic Survey.

The book might be improved considerably by numbering the formulas and by making the important ones stand out more prominently than is done in the text. For example, the equation for a meridian in the general polyconic projection is derived on page 12 without comment as to its basic importance, indeed without stating that it is the equation for a meridian.

Those who are interested in the practical construction of maps will no doubt find the book of great assistance, and to these it must make its appeal.

L. W. DOWLING.

Einleitung in die Mengenlehre. By A. Fraenkel. Berlin, J. Springer, 1919. iv + 155 pages.

The author proposes to give an introduction to the theory of infinite sets which can be understood by anyone who has sufficient interest and patience. No other prerequisites are set down. The author tells that he had experience in this sort of presentation during the recent war, when he lightened many wearisome hours by explaining *Mengenlehre* to his comrades in the field.

Among the chapter headings are: the concept of set; the concepts of equivalence and infinite sets; countable sets; the continuum; the concept of cardinal number; comparability of cardinal numbers; operations on cardinal numbers; ordered sets and types of order; linear point sets; well-ordered sets, well-ordering and its significance; logical paradoxes and the concept of set.

The choice of topics and the extent to which each topic is treated are well determined. Scientific honesty is not sacrificed