PLEASANT QUESTIONS AND WONDERFUL EFFECTS.

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This is for me a day of reckoning. For us mathematicians most of our days are days of reckoning. But usually, to revive an old phrase, we reckon without our host, without consideration of what we owe and to whom we owe it. I feel that I ought at least to thank the host and to contemplate the question of payment. And at once one sees various hosts.

First and foremost the host is this Society, one of the strongholds of idealism in this country. It was a most happy thought when Columbia men formed the scattered pools of mathematical activity into this important organization. No pool suffered a loss of what may be called potential, many gained enormously. The price must have been paid in the increasing work and sacrifice of the early officers. We took what they provided in the cheerful way of youth. And the youthfulness of the mathematician outlives that of most men, at least if he attends a fair number of meetings of this Society. For he can unload his mind if it is overburdened with a problem too hard for him. And he is sure to go away with some idea, grandiose or neat, obtained in that way of chalk and talk which is the easiest way of getting ideas, to a first approximation.

Or, secondly, the host to whom gratitude should be expressed may be the Engelschaar of great men to whom one owes the science as it stands. But this is obviously too big a field. An interesting point here is the effect of unreasoning veneration on elementary teaching, for example the effect of Euclid on elementary geometry or of Euler on elementary algebra. The adjustment of the properly conservative tradition of teaching to admit important applications is no easy matter. But a good guide in the elementary teaching of any subject is the consideration of its immediate usefulness in neighboring regions, its power of trespass.

Thus arithmetic is rightly taught commercially. Algebra is on the one hand universal arithmetic, on the other hand it is properly limited by its usefulness in geometry. Geometry should be taught with reference to its many uses. Thus every theorem would have an ultimate root in the ground of intelligent human interest. It would have as its motto "Et documenta damus qua simus origine nati."

The attempt to introduce any branch of mathematics as a pure self-contained logical science is bound to disappoint. I do not, of course, mean this to apply to non-elementary teaching. It is proper to separate the sheep from the goats, but I believe it is a mistake to separate the lambs from the kids.

Or, thirdly, the host to whom one is in private duty bound might be the ideas themselves which caught one's mind and helped to form it. These ideas will be, according to taste, logical or musical or practical. For myself, I confess gratitude to certain simple aids like the chess board or the meridians and circles of latitude on a sphere. With the simple background of a chess board in one's mind, one could survive the onslaught of the rabid algebraists and take a sane interest in determinants and in the elliptic functions of those days.

I have gratitude also for the secondary textbooks in that they gave some small point, some footnotes. You saw where a trail left the high road, where adventure began. This helped you on till you met the books which, being outside the curriculum, seemed pure adventure, as for instance, Whitworth's *Choice and Chance* or Casey's *Sequel to Euclid,* or Clifford's *Kinematics,* or the magnificent works of Salmon. Even the careful, learned, and not over-imaginative Todhunter would give hard and stimulating problems. With an attractive problem one hunted through the theory for a means of solution.

Thus the abstract theory of integration might leave one cold, but the finding of area, or centroid, or length of a curve, was a game. This is, I suppose, what lawyers call the case method, but it is just plain normal mental action.

Soon one saw that most decent theories had their applications, and one would take a chance on theory first. Or one would encounter at college a subject like rigid dynamics, where the problems were hurled at one beyond all reason.

It is possible to combine a reverent study of a reasonable number of classics with a decent development of one's own fragment of mind, to combine a respect for cathedrals with a liking for making mudpies; but for normal human beings the mudpie is the necessary first step to designing a cathedral or to respecting those who do. And our science needs its fair share of the normal-minded, to spread its ideas, to write its popularizing books. It is becoming too narrowly professional. There are not too many memoirs, but too few readable books.

Or, fourthly, the host may be those who have use for mathematics, the physicist or the statistician. Naturally, they would approve of the less formal introductory teaching which I have advocated. A few clear notions of applied mathematics acquired early will bear a fine superstructure, whereas one who tackles the mathematical treatises on (say) electricity, armed merely with a little analysis and less geometry, has a heart-breaking task if he aims at a real mastery of the mathematical side. So that the infusion of concrete applications, whether it is pedagogically sound as I asserted or not, is indispensable if we are to have a school of applied mathematicians. And this I hold to be a chief desideratum.

In this country research in pure mathematics is solidly rooted and actively efflorescent. Next comes the selection and use of the utilizable. This is the work of the applied mathematician. He and the mathematical physicist are on opposite sides of a counter in a drugstore. The latter seeks from the pharmacopeia what he needs; the former, knowing what the store contains, hands him usually not what he asks for, but a guaranteed article just as good. If more connection can be made between the ordered sense of beauty and power of calculation of the mathematicians and the strong intuitive grasp of our physicists and engineers, it is certain that wonderful effects will follow.

Fifth among the hosts should be the one subject which a man, now supposed to be grown, selects as his intellectual home. I venture to speak briefly for geometry on the ground of having worked diligently at relatively simple geometric questions.

That it should be treated logically is all right; but do not vivisect it merely for the sake of logic. By all means let it have foundations; but few will enter thereby. For it is properly the great exemplar of applied mathematics. And thanks to many workers, of whom Einstein is the latest and the most conspicuous, its future will be glorious.

The covariants and contravariants in which the projective and inversive geometers have delighted, but which they have signally failed to popularize, are now to be almost a commonplace in the wider field of differential geometry. The physicists who would not look at the projective gnats will swallow the differential camels. And there is no doubt more nutriment in a camel. There will be a lively market, and it should be met by some recasting where pedagogic reasons already exist. The mapping of spaces should be led up to by the simplest cases of mapping. Once again, what pedagogy sighed for physical science demands, this time in the field of elementary geometry itself.

If the science should be taught in its early stages not as a jumble of special applications, but always with an honest consideration of its legitimate contexts, then would it still be true of the far wider mathematics of today, that, to quote old Isaac Barrow again, "The Mathematics is the unshaken Foundation of Science and the Plentiful Fountain of Advantage in Human Affairs."

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FALLACIES AND MISCONCEPTIONS IN DIOPHANTINE ANALYSIS.

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§ 1. *Introduction.* Numerous writers have claimed to find all integral solutions of various homogeneous equations when they have actually found merely the rational solutions, expressed by formulas involving rational parameters. They have really left untouched the more difficult problem of finding all the integral solutions exclusively. The fallacies exposed in § 2 and § 3 are merely particular instances of the wide-spread misconception of the problem of solving a homogeneous equation in integers. It is therefore not safe, without reexamination, to place confidence in any claim that a homogeneous equation has been completely solved in integers.

In the next number of this BULLETIN, I shall show how the