$$- \frac{1}{4r}[v''' + (6q_2 - 2r)v' + 3q_2'v]_{z=\zeta}u_1(z)w_2(z)$$

$$+ \frac{1}{4r}[v''' + (6q_2 + 2r)v' + 3q_2'v]_{z=\zeta}u_2(z)w_1(z)$$

$$+ \tfrac{1}{2}[v''(\zeta) + 3q_2(\zeta)v(\zeta)]u_2(z)w_2(z).$$

$S(v, v) = 0$ is therefore the necessary and sufficient condition that $v(z)$ be factorable as the product of a solution of (5) by a solution of (6). Therefore, if $S(v, v) = 0$ the number of zeros of $v(z)$ in a given closed interval (a, b) will be limited by Sturm's theorems concerning the zeros of solutions of (5) and (6). If, for example, the equations (5) and (6) have k_1 and k_2 for their respective indices of oscillation in (a, b) then every solution of (5) [(6)] will have k_1 or $k_1 + 1$ [k_2 or $k_2 + 1$] zeros in the interval (a, b), and every solution, $v(z)$, of (3) for which $S(v, v) = 0$ will have $k_1 + k_2$, $k_1 + k_2 + 1$, or $k_1 + k_2 + 2$ zeros in (a, b). If, in particular, $q_2(z) < 0$ and $0 < C < 9q_2^2(z)$ throughout (a, b) then the coefficients of $u(z)$ and $w(z)$ in (5) and (6) will be negative throughout (a, b) and $k_1 = k_2 = 0$.* Therefore, *if* $q_2(z) < 0$ *and* $0 < C < 9q_2^2(z)$ *throughout* (a, b) *then no solution of* (3) *for which* $S(v, v) = 0$ *has more than two zeros in* (a, b), *and some such solutions have no zeros in* (a, b).

WESLEYAN UNIVERSITY,
MIDDLETOWN, CONN.

AN ACKNOWLEDGMENT OF PRIORITY.

BY PROFESSOR A. A. BENNETT.

IN the July, 1919, number of the BULLETIN (volume 25, page 455), I gave a solution condensed to about two pages, of a problem proposed by the Philosophical Faculty of the University of Berlin as a prize problem. My attention has been called to the fact that this question is completely handled in an analogous manner by Harold Hilton, in his book on "Homogeneous Linear Substitutions," 1914, in Chapter VI, section 8. Similar results are given in the same chapter for

* See Bôcher's Méthodes de Sturm, Paris, 1917, p. 52.

symmetric and Hermitian matrices. This book, while familiar to the American public, was not available to me at the time, and I did not realize that the problem proposed had been completely settled previous to the occurrence of the war.

February 9, 1920.

DICKSON'S HISTORY OF THE THEORY OF NUMBERS.

ON page 130 of my review of Dickson's History I speak of two pages of titles at the end of Chapter XII as "not reported on." This is an error. With the exception of certain ones marked with an asterisk which give papers not obtainable by the author the content of the papers is sufficiently indicated, the papers not being of importance to warrant more detailed report.

The phrase "list of references," on the top of page 132, is perhaps misleading. Of course the book is in no sense to be compared with the useless lists of titles of papers which the compiler may or may not have glanced at. My intention in the paragraph was to bring out the distinction which modern historians seem to make between a list of events and the relations and connections between events.

D. N. LEHMER.

NOTES.

THE December number (volume 21, number 2) of the *Annals of Mathematics* contains the following papers: "A property of cyclotomic integers and its relation to Fermat's last theorem," by H. S. VANDIVER; "Surfaces of rotation in a space of four dimensions," by C. L. E. MOORE; "The circle nearest to n given points, and the point nearest to n given circles," by J. L. COOLIDGE; "Singular solutions of differential equations of the second order," by E. M. COON and R. L. GORDON; "Note on a class of integral equations of the second kind," by C. E. LOVE; "Concerning sense on closed curves in