as a $(2n-1)$-space tangent to the n-dimensional quadratic cone K_n' of $(n-2)$-spaces also represented as Q_n'. While a P' of Q_n' meets Q_n in a conic, the two $(2n-2)$-spaces, L', tangent to K_n' and contained in P', which are also tangent to Q_n, determine two points of tangency on Q_n. This correspondence is again two-two, and for it the same theorem holds. The case $n = 2$ leads to the study of Kummer's surface and the theorem is in substance familiar in this case. Cf. Hudson, Kummer's Quartic Surface, Cambridge, 1905, page 196, and Zeuthen, Lehrbuch der abzählenden Methoden der Geometrie, Leipzig, 1914, page 276.

WASHINGTON, D. C.,
November, 1919.

ON THE RECTIFIABILITY OF A TWISTED CUBIC.

BY DR. MARY F. CURTIS.

(Read before the American Mathematical Society December 31, 1919.)

IF the space curve

$$x_i = a_i t^n + b_i t^{n-1} + \cdots + k_i t + l_i \quad (i = 1, 2, 3) \tag{1}$$

is a helix, it is algebraically rectifiable. For if it is a helix, it makes with a fixed direction a constant angle and $\sqrt{x'|x'} = (x'|\alpha)$,* where $\alpha_1, \alpha_2, \alpha_3$ are constants, not all zero; then the arc

$$s = \int_{t_0}^{t} \sqrt{x'|x'}\, dt \tag{2}$$

is an integral rational function of t, not identically zero, and the curve (1) is algebraically rectifiable.

It is not, however, in general true, that if (1) is algebraically rectifiable, it is a helix. It will be true, provided (2) is an algebraic function only when $(x'|x')$ is a perfect square of the form $(x'|\alpha)^2$. This condition is fulfilled in the case of the twisted cubic:

$$x_1 = at, \qquad x_2 = bt^2, \qquad x_3 = ct^3, \qquad abc \neq 0, \tag{3}$$

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* If $a : (a_1, a_2, a_3)$ and $b : (b_1, b_2, b_3)$ are two triples, then by $(a | b)$ we mean their inner product: $a_1b_1 + a_2b_2 + a_3b_3$.

and hence, as I pointed out in a note on page 87 of volume 25 (1918) of this BULLETIN, the cubic (3) is algebraically rectifiable when and only when it is a helix.

As a generalization of this result Professor Tsuruichi Hayashi, in the November number of the current volume of the BULLETIN, has for the twisted cubic

$$(4) \quad x_i = a_i t^3 + b_i t^2 + c_i t + d_i \qquad (i = 1, 2, 3, \quad |abc| \neq 0),$$

which, by a change of parameter and axes, he writes in the form

$$(5) \quad x_1 = a_1 t^3 + c_1 t, \quad x_2 = a_2 t^3 + b_2 t^2, \quad x_3 = a_3 t^3, \quad a_3 b_2 c_1 \neq 0,$$

the theorem: The conditions

$$(6) \qquad a_2 = 0, \qquad 9a_3^2 c_1^2 = 12 a_1 c_1 b_2^2 + 4 b_2^4,$$

are necessary and sufficient both that (5) be a helix and that (5) be algebraically rectifiable.

This theorem, as stated, is not true. The cubic (5) may be algebraically rectifiable if not a helix, for its arc (2), where

$$(x'|x') = 9(a|a)t^4 + 12 a_2 b_2 t^3 + (6 a_1 c_1 + 4 b_2^2) t^2 + c_1^2,$$

is an algebraic function, not only when $(x'|x')$ is a perfect square but also when $(x'|x')$ is a cubic with a multiple root, namely, when

$$(7) \quad a_2 \neq 0, \quad (a|a) = 0, \quad 2(3 a_1 c_1 + 2 b_2^2)^3 + 243 a_2^2 b_2^2 c_1^2 = 0.$$

Hence Professor Hayashi's theorem is true for the cubics (5) only if those for which (7) holds are excluded.

Since the cubics (5) for which (7) holds are imaginary, we may say: *A real twisted cubic* (5) *is algebraically rectifiable, when and only when it is a helix.*

It is interesting to note that the facts for the cubics given by (5) do not quite correspond to those for the cubics given by (4), from which (5) was deduced, because, in the reduction of (4) to (5) certain imaginary cubics given by (4) have been excluded; among them are the helices, for which $(x'|x')$ for (4) reduces to the form: $(kt^2 + lt)^2 \neq 0$, and the algebraically rectifiable cubics, not helices, obtained when $(x'|x')$ for (4) is of the form: $mt + n$, $m \neq 0$. But, as these curves are all imaginary, the theorem just stated for the cubics (5) holds equally well for the cubics (4).

In order that the general twisted cubic (4) be algebraically rectifiable, it is sufficient, but not necessary, that it be a helix. By demanding algebraic rectifiability of a particular type, we may obtain a condition in terms of the curvature and torsion of (4), which is both necessary and sufficient:

The general twisted cubic (4) *may be represented by equations of the form* (4), *where t is its arc, when and only when it is of constant curvature and torsion.*

For a necessary and sufficient condition that (4) may be so represented is that $(x'|x')$ is a constant, not zero, and this in turn can be shown to be necessary and sufficient that (4) is a curve of constant curvature and torsion.

WELLESLEY COLLEGE,
November 15, 1919.

NOTE ON LINEAR DIFFERENTIAL EQUATIONS OF THE FOURTH ORDER WHOSE SOLUTIONS SATISFY A HOMOGENEOUS QUADRATIC IDENTITY.

BY DR. C. N. REYNOLDS, JR.

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IN this paper I shall prove that if a given homogeneous linear differential equation of the fourth order has four linearly independent real solutions which satisfy a non-singular homogeneous quadratic identity, then it may be reduced to a form which may be said to be self-adjoint with respect to the third row of the wronskian of any four linearly independent solutions. I shall then evaluate the signature of the quadratic identity in terms of the coefficients of the reduced equation and derive several theorems concerning the zeros of solutions of such equations.

Brioschi* has shown that if $y_1(x)$, $y_2(x)$, $y_3(x)$ and $y_4(x)$ are four linearly independent real solutions of the equation

$$(1)\quad \begin{aligned} p_0(x)y^{\text{IV}}(x) &+ 4p_1(x)y'''(x) + 6p_2(x)y''(x) \\ &+ 4p_3(x)y'(x) + p_4(x)y(x) = 0 \quad (p_0(x) \neq 0), \end{aligned}$$

* "Les invariants des équations différentielles linéaires," *Acta Mathematica* (1890), vol. 14, p. 233.