PONCELET POLYGONS IN HIGHER SPACE.

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LET there be given a linear projective space of $2n$ dimensions. A point of the space may be denoted by P and its dual figure by P'. Thus a P' is a linear space of $2n - 1$ dimensions.

The totality of P's in the space is infinity to the order $2n$, and the totality of P''s is of course of this same order. We shall select from these totalities a Q_n and a Q_n' respectively, general quadratic loci of infinity to the order n of elements, where Q_n consists of P's, and Q_n' of P''s.

For Q_n and Q_n' not in specialized relation to each other we have a two-two correspondence of the following form: Each P of Q_n meets two P''s of Q_n', and each P' of Q_n' meets two P's of Q_n. Starting with any point of Q_n, a succession of points of Q_n is determined, where furthermore consecutive points of the sequence may be joined by lines. The succession of lines forms then a single "broken line" as this term is used in projective geometry. It may or may not happen that the broken line closes into a polygon. Except for degenerate cases corresponding to coincident P's or P''s, and it being supposed that Q_n and Q_n' are not degenerate, it may be proved that the closure of the broken line is determined by the relative positions of Q_n and Q_n' and is independent of the element selected as initial.

This may be called a theorem of Poncelet polygons in higher spaces. For $n = 1$, the theorem is the usual one.

It should be emphasized that the case for $n > 1$ is not the logical equivalent of the case for $n = 1$, since there are n independent parameters in any case. The proof of the theorem is immediate by reference to general theorems on algebraic correspondences or to theta functions, the quadratics Q_n and Q_n' determining theta functions of genus n, and affording one of the simplest illustrations of their character.

A second generalization and one which applies to three-space is to spaces of $2n - 1$ dimensions generally, $n > 1$, the P, P', Q_n, Q_n' being as above. Any P' of Q_n' may be viewed as a $(2n-1)$-space tangent to the n-dimensional quadratic cone K_n' of $(n-2)$-spaces also represented as Q_n'. While a P' of Q_n' meets Q_n in a conic, the two $(2n-2)$-spaces, L', tangent to K_n' and contained in P', which are also tangent to Q_n, determine two points of tangency on Q_n. This correspondence is again two-two, and for it the same theorem holds. The case $n = 2$ leads to the study of Kummer's surface and the theorem is in substance familiar in this case. Cf. Hudson, Kummer's Quartic Surface, Cambridge, 1905, page 196, and Zeuthen, Lehrbuch der abzählenden Methoden der Geometrie, Leipzig, 1914, page 276.

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ON THE RECTIFIABILITY OF A TWISTED CUBIC.

BY DR. MARY F. CURTIS.

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IF the space curve

$$x_i = a_i t^n + b_i t^{n-1} + \cdots + k_i t + l_i \quad (i = 1, 2, 3) \tag{1}$$

is a helix, it is algebraically rectifiable. For if it is a helix, it makes with a fixed direction a constant angle and $\sqrt{x' | x'} = (x' | \alpha)$,* where $\alpha_1, \alpha_2, \alpha_3$ are constants, not all zero; then the arc

$$s = \int_{t_0}^{t} \sqrt{x' | x'}\, dt \tag{2}$$

is an integral rational function of t, not identically zero, and the curve (1) is algebraically rectifiable.

It is not, however, in general true, that if (1) is algebraically rectifiable, it is a helix. It will be true, provided (2) is an algebraic function only when $(x' | x')$ is a perfect square of the form $(x' | \alpha)^2$. This condition is fulfilled in the case of the twisted cubic:

$$x_1 = at, \qquad x_2 = bt^2, \qquad x_3 = ct^3, \qquad abc \neq 0, \tag{3}$$

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* If $a : (a_1, a_2, a_3)$ and $b : (b_1, b_2, b_3)$ are two triples, then by $(a | b)$ we mean their inner product: $a_1b_1 + a_2b_2 + a_3b_3$.