

schools of all the world. More than ever it is incumbent upon American leaders in education to exert every effort to improve the preparation of the teachers and the status of the profession, for it is to our system of education that Europe will turn for guidance.

The recent tremendous increases in enrollment of students in our universities includes fortunately large numbers who are taking up the study of mathematics. It is somewhat significant that this increase is affecting only the universities, not the normal schools. Undoubtedly, the day is not distant when our universities and colleges will be called to train not only high school teachers, but also the "junior high school" teachers. The nature of this training for the instructors in mathematics in the secondary schools needs systematic and continued study, taking into account the changes in the world in which we live.

This study under review, while reflecting a world which has passed, will nevertheless continue for many years to be of great value to all concerned with the preparation of our teachers of mathematics.

L. C. KARPINSKI.

Theory of Maxima and Minima. By HARRIS HANCOCK.
New York, Ginn and Company, 1917. v + 193 pages.
Price \$2.50.

THE student of mathematics meets the interesting subject of maxima and minima early in his first course in calculus. The subject appears again and again in his courses in advanced calculus and functions of a real variable, each time carrying the student a little deeper into the theory, but rarely giving him opportunity to view the subject as a whole. The English reading student finds it particularly difficult to study the theory for functions of two or more variables as expounded by Scheefer, Stolz, von Dantscher, and Weierstrass. Professor Hancock's book is for this English reading student.

The opening chapter discusses functions of one variable, first taking up functions having complete derivatives throughout the interval in question (ordinary maxima and minima). This covers the usual discussion in a first course in calculus with a somewhat more mature reader in view. Then follows a discussion for functions having derivatives only for definite values of the variable, or having one-sided derivatives (ex-

traordinary maxima and minima). A statement of the theorem concerning the existence of the upper and lower limits of a continuous function in a definite interval is included without proof.

Chapter II is a preliminary chapter on functions of several variables, outlining in a broad way the general problem. Chapter III is limited to functions of two variables, treating in a general way the different cases that may arise, pointing out the difficulties of the ambiguous case, discussing the errors that are in the literature and noting the various attempts to improve the theory. This leads up to Chapter IV in which Scheefer's theory for two variables is developed in detail with a general outline of Stolz's extension to the case of three variables. Von Dantscher's theory is included for comparison purposes. These three chapters are the most important in the book. They give in sufficient detail the theorems of Scheefer, Stolz and von Dantscher together with a discussion of the errors made by earlier writers and carried on into many of the modern textbooks on the calculus. In particular the so-called ambiguous case, around which most of the difficulties center, is treated in a fuller manner than in any other work in English that the reviewer has examined. On page 34, the author quotes from Professor Pierpont's article in Volume IV of the *BULLETIN*, "Our English and American authors seem to be ignorant of these facts." At first the reviewer was inclined to look upon this quotation written in 1898 as rather historical, but a study of the English and American books available failed to bring forth a full discussion of the maxima and minima of functions of two variables. However in many cases the reason was not that the authors seemed "to be ignorant of these facts," but simply that they lacked the space for a full treatment. This full treatment is here.

The last four chapters in the book follow very closely the work published by Professor Hancock in 1903, *Lectures on the Theory of Maxima and Minima of Functions of Several Variables (Weierstrass' Theory)*.

These chapters, which follow the lectures of Weierstrass, treat only the ordinary cases where the functions are everywhere regular and where the forms are either definite or indefinite, excluding the semidefinite form characterizing the ambiguous case. Some interesting additions have been made

to the older work. Among them are two interesting light theory problems prepared by Professor Brand. One problem is concerned with the necessary and sufficient conditions for minimizing the length of a ray of light from one point to another after reflection at a point on the surface $F(x, y, z) = 0$. The other is a refraction problem in which the time between two points is to be a minimum. Hadamard's determinant problem is also included.

Throughout the book are many examples and problems illustrating points in the theory, some worked out in detail.

A. R. CRATHORNE.

From Nebula to Nebula, 4th Edition. By GEORGE HENRY LEPPER. Pittsburgh, Pa., privately printed, 1919. 401 pages.

(FROM the introduction) "The object of this work is twofold: (1) to present a revaluation of the time-honored doctrines upon which modern theoretical astronomy is based, and (2) having shown wherein they are defective, to propose a new and far more comprehensive system, revealing the entire visible universe in the philosophic aspect of a single unit coordinated throughout, as *a priori it must be*, by a single dynamical force."

It is not possible, within the limits of available space, to give a complete analysis of this book. A few quotations will indicate some of the author's conclusions, but the proofs which he gives must be omitted. These quotations are so chosen that the lack of context need do no injustice to the author.

"(The earth . . .) is generously endowed with liquid oceans, which she is compelled to shift constantly from the sunlit side in her efforts to preserve her center of gravity at the lowest point. It is this struggle for equilibrium . . . which I assign as the major cause of the earth's diurnal rotation."

"I contend, with Aristotle, that rest is the natural state of matter."

"I have been enabled . . . to demonstrate the sun's path to be that of an immense spiral with a diameter of 1,530,000 million miles and a total coil length of nearly seven trillions."

"Paradoxical as it may sound, . . . I contend that the sun and not the moon is the *vera causa* of the tides."

After attempting to prove that tides cause the rotation of