THE OCTOBER MEETING OF THE SAN FRANCISCO SECTION.

THE thirty-fourth regular meeting of the San Francisco Section was held at the University of California on Saturday, October 25. There were two sessions. The morning session was opened by the chairman of the Section, Professor Cajori, who later was relieved by Professor Blichfeldt.

The attendance was twenty-one, including the following fourteen members of the Society:

Professors R. E. Allardice, B. A. Bernstein, H. F. Blichfeldt, Thomas Buck, Florian Cajori, M. W. Haskell, L. M. Hoskins, D. N. Lehmer, W. A. Manning, H. C. Moreno, Dr. F. R. Morris, Professors C. A. Noble, T. M. Putnam, and Dr. Pauline Sperry.

The following officers were elected for the year: chairman, Professor H. F. Blichfeldt; secretary, Professor B. A. Bernstein; programme committee, Professors W. A. Manning, D. N. Lehmer, and B. A. Bernstein.

The dates of the next two meetings were fixed as April 10, 1920, and October 23, 1920.

The following papers were presented:

(1) Professor W. A. MANNING: "Doubly transitive groups with transitive subgroups of lower degree."

(2) Professor M. W. HASKELL: "On self-dual curves" (preliminary report).

(3) Professor W. A. MANNING: "Groups of degree n in which there is a circular permutation of n letters."

(4) Professor FLORIAN CAJORI: "Surveying and astronomical instruments used in America before the nineteenth century."

(5) Professor E. T. BELL: "On a certain inversion in the theory of numbers."

(6). Professor E. T. BELL: "On the enumeration of proper and improper representations in homogeneous forms."

(7) Professor E. T. BELL: "On proper and improper representations in certain quadratic forms of Liouville."

(8) Professor R. M. WINGER: "Some generalizations of the satellite theory."

In the absence of the authors the papers of Professors Bell and Winger were read by title. Abstracts of the papers follow below.

1. Professor Manning presented the following theorem:

Let a doubly (but not triply) transitive group of degree n have transitive subgroups $H, H_1, \cdots, H_r$ of degrees $m, m + q_1, \cdots, m + q_1 + \cdots + q_r$, respectively, and of no other degree $< n$ and $> m$. Then H_{i-1} ($i = 1, 2, \cdots, r$; $H_0 = H$) has at least

$$1 + (q_i + q_{i+1} + \cdots + q_u)\left[\frac{1}{q_{u+1}} + \frac{1}{q_{u+2}} + \cdots + \frac{1}{q_r} + 1\right]$$

systems of imprimitivity of q_u letters which have one letter in common ($u = i, i + 1, \cdots, r$). When $u = r$, no two of the above systems of q_r letters that have one letter in common have a second letter in common.

In particular this theorem asserts that H has $1 + q_1 + q_2 + \cdots + q_r$ systems of q_r letters with one letter in common.

This problem was originated by C. Jordan. He showed (1871) that H_r, and a fortiori $H, H_1, \cdots$, must have at least two systems of imprimitivity of q_r letters with one letter in common. Then B. Marggraff (1889) proved that H_r has $1 + q_r$ systems of imprimitivity of q_r letters with one letter in common. This result, as Marggraff did not fail to notice, reveals the presence in H_{i-1} of at least $1 + q_u/q_{u+1}$ systems of q_u letters with q_{u+1} letters in common ($u = i, i + 1, \cdots, r - 1$). It was subsequently shown (Manning, 1906) that systems of imprimitivity of H_{i-1} of q_u letters each, with q_{u+1} letters in common, can be chosen in at least

$$1 + q_u\left[\frac{1}{q_{u+1}} + \frac{1}{q_{u+2}} + \cdots + \frac{1}{q_r} + 1\right]$$

ways ($u = i, i + 1, \cdots, r$). The corresponding result, as stated in the present theorem, was suggested by a study of the group of isomorphisms of the elementary group of order 2^n.

2. Professor Haskell gave a preliminary account of the self-dual curves which are self-reciprocal with respect to a conic. He showed that any conic is self-reciprocal with respect to

each of a doubly infinite set of conics, and that this relation is mutual; that the self-dual cubic is self-reciprocal with respect to a singly infinite set of conics; and that the quintic with five cusps is self-reciprocal with respect to a single conic.

3. It was proved by Burnside that a simply transitive group of degree p (p a prime) is cyclic or contains an invariant subgroup of order p. He also proved that a simply transitive group of degree p^n in which there is a permutation of order p^n is necessarily imprimitive and compound. In his second paper Professor Manning announced the more general theorem:

If a simply transitive group of composite order contains a circular permutation of all its letters, it is compound.

4. Professor Cajori pointed out that the surveying and astronomical instruments used in America before 1800 were almost altogether from English makers—Heath, Short, Nairne, Bird, Sisson, Dollond, Ramsden. The chief American instrument makers were David Rittenhouse and Thomas Godfrey. Photographs of instruments were exhibited.

5. The inversion considered in Professor Bell's first paper is as follows. Define $f(x)$ to be regular if it exists and has a finite value when x is an integer > 0, and $f(1) \neq 0$. Then there exists a unique regular f', called the inverse of f, such that

$$\Sigma f(d)f'(n/d) = f(1) \text{ or } 0$$

according as $n = 1$ or $n > 1$, the Σ extending to all divisors d of the arbitrary integer $n > 0$. The f' thus defined is distinct from Berger's inverses. The theorem, which in some respects is similar to the well known inversion of H. F. Baker, is useful in many parts of arithmetic. The paper will appear in the *Tôhoku Mathematical Journal.*

6. Algebraic processes, elliptic function or other, do not easily yield the number of proper representations of an integer in a homogeneous form, but when applicable at all, give the total number of representations with great readiness. In the arithmetical treatment, it has been customary to deduce the total number of representations from the number of

proper representations. Inverting this procedure, Professor Bell, in his second paper, proves formulas of remarkable simplicity for the expression of the proper number in terms of the total, so that if the latter is within reach of analysis, so also now is the former. A few illustrations are given in the derivation of new results concerning 6, 8, 10 or 12 squares and other simple quadratic forms. The cases of 10, 12 squares present some unexpected singularities.

7. From 1860 to 1864 Liouville published in his *Journal* numerous theorems on the number of total and proper representations of integers in special quadratic forms of four and six indeterminates. His formulas for total numbers of representations were for the most part proved in 1890 by Pepin, those omitted being readily demonstrable by elliptic functions and other algebraic means. The formulas for proper representations have not hitherto been proved. Modifying the general principles of his second paper to fit Liouville's forms, Professor Bell demonstrates all of the unproved results very simply. The paper will appear in the *Journal de Mathématiques pures et appliquées*, (in French), and a fuller abstract shortly in the Paris *Comptes Rendus*.

8. Professor Winger's paper appeared in full in the November BULLETIN.

B. A. BERNSTEIN,
Secretary of the Section.

ON THE PROOF OF CAUCHY'S INTEGRAL FORMULA BY MEANS OF GREEN'S FORMULA.

BY MR. J. L. WALSH.

(Read before the American Mathematical Society December 30, 1919.)

IT is well known that Cauchy's integral formula for an analytic function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$ is analogous to Green's formula for the functions u and v, and that moreover Cauchy's formula can be proved from Green's formula. Picard (Traité d'Analyse (1905), volume II, page 114) gives this proof *assuming that*