In fact, the book does not at all help the student to realize the ideal expressed by the author, that "the student should at all times strive to get answers as nearly accurate as the tables will permit." Nothing whatever is said about the degree of accuracy warranted by the given statistics of a problem.

Errata are numerous in the text. Some parts of the book are poorly written. There is a dearth of punctuation in general, and of periods in particular. In places the arrangement of the material is very poor, while in other portions of the text the material is well spaced and the page presents its salient points at a glance. The figures and curves are very good.

<div style="text-align: right">M. E. WELLS.</div>

Tables numériques usuelles. Par L. ZORETTI. Paris, Gauthier-Villars, 1917. 8vo. 52 pp. 3 fr.

THESE small tables are designed for the use of engineers, students of college and lower grades, and general computers. They consist of multiplication tables for the multipliers 1, 2, $\cdots$, 9 and multiplicands of three figures; together with columns that give for three figure arguments the reciprocal, the square and the common logarithm to four places. A table of trigonometric functions, grades, and radians for every degree and quarter degree follows. Tables of logarithms of certain common constants, and the values of πd and $\frac{1}{4}\pi d^2$ for two figure arguments close the volume. The leaves are thumb-indexed and by proper spacing made otherwise convenient. The type is clear and the paper with no glare. The book is ample for most of the computations involved in the author's general course in mathematics.

<div style="text-align: right">JAMES BYRNIE SHAW.</div>

Ueber gewisse Teilbereiche und Erweiterungen von Ringen. By Dr. Phil. ADOLF FRAENKEL. Leipzig und Berlin, Teubner, 1916. 64 pp.

IN the investigations of the arithmetic properties of the algebraic numbers the fundamental notion is that of domain of rationality, in which the four operations of arithmetic, excluding only division by zero, are permissible. Besides these certain other sets, called orders by Dedekind and rings

by Hilbert, have been used. In these rings the operations addition, subtraction, and multiplication are permissible, but division is not always possible and, when possible, does not always lead to a unique result.

These rings are special types of groups, where the term group is used in the more general sense of the French edition of the mathematical encyclopedia. The first general investigation into their nature was made by the author of the present work in his dissertation, published in *Crelle's Journal*, volume 145. In this investigation he makes frequent use of the group property and in the proofs of some theorems he shows that the elements under consideration form a group in the ordinary sense and draws his conclusions from known theorems in the theory of groups.

The present work is a further study of certain types of rings, with special reference to certain subsets, and extensions formed by the adjunction of new elements.

Two elements a and b of a ring R are called equivalent if a is divisible by b and b by a. A divisor of zero is an element b such that $a \cdot b = 0$ when $a \neq 0$. An element which is not a divisor of zero is a regular element. A divisor of zero which, aside from regular elements, has no divisor except itself is called a prime divisor. A simple ring is one which contains only one prime divisor.

The work is devoted to the consideration of rings in which multiplication is commutative and in which the following two conditions are satisfied:

I. There are only a finite number of non-equivalent divisors of zero.

II. If $a \cdot b$ is divisible by c, then there exist two elements c_1 and c_2 such that $c = c_1 \cdot c_2$ and a is divisible by c_1 and b by c_2.

Rings which satisfy these conditions are called decomposable, because from the elements of such a ring R, in which there are n non-equivalent prime divisors, we may construct n simple rings $R_1, R_2, \ldots, R_n$, such that no element of R, except zero, is contained in more than one of the rings R_i, the product of two elements from two different rings R_i and R_j is zero, and every element of R has a unique representation as the sum of n elements, one from each of the rings R_i.

This property is of fundamental importance in the theory of the g-adic numbers which constitute a ring such as has been defined above.

A partial ring $\overline{R}$ of a ring R is a set of elements of R, closed with respect to addition, subtraction and multiplication, but in which the equation $ax = b$ has a solution only when it has a solution in R. Since the author has chosen to call a ring a set of elements in which division by regular elements is always possible and a may be a divisor of zero in R but regular in $\overline{R}$, we see that $\overline{R}$ is not necessarily a ring.

A prime ring is a partial ring which does not contain as a subset another partial ring having the same unit element.

The special prime ring is that prime ring whose unit element is the unit element ϵ of R.

A part of the first chapter is devoted to the determination of the structure of the special prime rings. They are found to be of four types according as the sequence $\epsilon, 2\epsilon, 3\epsilon, \ldots$ contains: (1) neither zero nor divisors of zero; (2) zero but no other divisor of zero; (3) both zero and divisors of zero; (4) divisors of zero but not zero.

The concluding section of the chapter deals with prime rings in general and contains a theorem regarding the number of prime rings in a given ring and their structure.

To aid the reader in understanding the theory the author gives, at various points in the chapter, examples of rings chosen from systems of residues with respect to a certain modulus or from systems of complex numbers.

Chapter II consists of a study of the rational functions of x in a simple ring R. The degree of a polynomial is defined as usual. The order of a polynomial is the exponent of the highest power of x whose coefficient is a regular element of R. The degree of the product of two polynomials may be less than the sum of their degrees, but the order of the product is the sum of the orders.

A polynomial which has at least one regular coefficient is called a primitive polynomial. Every primitive polynomial of order n can be multiplied by a primitive polynomial of order zero so that the product will be of degree n. By applying this theorem the author discusses Euclid's algorithm in a simple ring. This process always leads to the highest common divisor if the successive remainders are all primitive, but fails if the process finally gives a remainder which is not primitive.

Chapter III contains a discussion of the extensions of a simple ring formed by the adjunction of new elements. An

extension formed by adjoining an element which satisfies an algebraic equation in R is called an algebraic extension. If the element which is adjoined does not satisfy an algebraic equation in R the extension is called transcendental.

From the simple transcendental extensions formed by the adjunction of the element x to R the author passes to the algebraic extensions by assigning to x a value which is a root of an algebraic equation.

Finally by making use of the theorem regarding the decomposition of rings the results found are extended to rings having more than one prime divisor.

G. E. WAHLIN.

Calcul des Systèmes Elastiques de la Construction. Par ERNEST FLAMARD. Paris, Gauthier-Villars, 1918.

THIS work is a treatise of 200 pages, published under the imprint of the "Encyclopédie Industrielle," on the application of the principle of least work to the calculation of the reactions and deflections of straight and curved beams, the elastic arch, and pin-connected structures having redundant members or supports.

The feature of most interest to the writer of this review is the evidence the work furnishes that there are mathematicians and engineers in France today of such mental poise that they are able to concentrate their attention on a purely theoretical question of method which brings out no new results and has absolutely no relation to the war or the future.

In this work it is first shown that the elastic forces acting on any section of a solid are reducible to three static elements consisting of a bending moment and normal and shearing forces. The expressions for the work of deformation due to these three elements are then derived, as well as Castigliano's well known theorem, giving the linear and angular displacements of the external forces and couples in terms of the partial derivatives of the work of deformation with respect to these elements. This is followed by the derivation of the principle of least work of deformation. These results are then extended to include the effect of change in temperature. In applying the results to beams under vertical loading, however, it is pointed out that the temperature forces are the only external forces acting parallel to the axis of the beam, and