

mais devant une tombe qui vient de se fermer, nous ne rappelons que son génie et cette universelle sympathie qui s'accorde à la noblesse du caractère. Weierstrass a été droit et bon; qu'il reçoive le suprême hommage plein de regrets et de respect que nous adressons à sa mémoire! Elle vivra aussi longtemps que des esprits avides de vérités consacreront leur efforts aux recherches de l'Analyse, au progrès de la science du Calcul."

Let us quote from one more éloge, that of Brioschi: "J'ai été associé aux travaux de Brioschi; nous avons souvent mis en commun nos efforts; j'ai suivi sa carrière qui a été si belle, remplie par l'étude, et de grands services rendus à son pays. Nul ne ressent plus que moi la perte du grand géomètre, et de l'homme d'honneur, le souvenir de son amitié, d'une étroite liaison remontant à notre jeunesse me restera à jamais l'un des meilleurs et des plus chers de toute ma vie."

Other interesting biographical matter is contained in an address by Hermite at the inauguration of the new Sorbonne in 1900. It gives a brief but illuminating analysis of the work of the chief French mathematicians in the first half of the nineteenth century.

Finally we note the touching discours pronounced by Hermite at the celebration of his seventieth anniversary in 1892. A photograph of the medallion by Chaplain presented to him on this occasion by his friends and admirers from all over the world forms the frontispiece of the volume and a facsimile reproduction of an extract of a letter to Tannery relative to Hermite's classic researches on the elliptic modular function in 1858 brings the volume to a fitting close.

JAMES PIERPONT.

## BLICHFELDT'S FINITE COLLINEATION GROUPS.

THE author of "Finite Collineation Groups" is certainly under great obligations to Professor H. H. Mitchell for his very generous and thorough review in this BULLETIN of February, 1918; particularly so since Professor Mitchell had evidently read the book with great care. The author is in full accord with the reviewer on a number of the defects pointed out by the latter; and he offers herewith a few remarks in the hope of clearing up those statements in the book that

seemed inexact or obscure to him upon reading Professor Mitchell's review. Unless otherwise stated, page and paragraph references are to the book in question.

1. (BULLETIN, page 247.) Kronecker's theorem was originally stated correctly by the author (*Transactions of the American Mathematical Society*, 1903, pages 390-391; *Mathematische Annalen*, 1907, page 558 (in this place, line 3 from the bottom, the inequality  $t < p$  should read  $t < p^{n-1}$ )). The incorrect form occurs in *Mathematische Annalen*, *ibid.*, page 559; in *Finite Groups* by Miller-Blichfeldt-Dickson, page 240; and in the book under discussion, page 187. After making the proper corrections in the sentence beginning "Kronecker's theorem implies" (page 187, lines 3-5), namely by substituting for " $k$  roots" the following: "the roots of this equation (either directly or after adding a certain number of pairs of mutually cancelling roots of unity)," the author would leave the next sentence unaltered, and then introduce a new sentence (line 8) to read: "In no case need  $p$  be taken greater than  $k$ ." The remaining portion of 6° will stand as it is. Under 7°, the sentence beginning (line 6 from the bottom) "More definitely" should be replaced by "In particular, if the total number of roots in  $N$  equals  $k$ , then all these roots are equal, or  $N = 0$ ; if the number of roots is  $< k$ , then  $N = 0$ ."

2. (BULLETIN, page 249.) In the chapter on group characteristics the meaning of the term "equivalent" is in accordance with the usage in the writings of Burnside, Frobenius, and Schur, though these authors say "equivalent representations of a group" instead of "equivalent groups." Two simply isomorphic groups having their operators so arranged that  $S_i$  in the one corresponds to  $T_i$  in the other,  $i = 1, 2, \dots, g$ , are equivalent if a change of variables (transformation)  $V$  exists such that  $V^{-1}S_iV = T_i$  for every subscript  $i$ . The same two groups may possibly not be equivalent when a new one-to-one correspondence is set up in a different manner. The author has, admittedly, used the word freely in the chapters on the binary, ternary and quaternary groups without calling attention to the fact that in these chapters, if the phrase "non-equivalent" were used in its strict sense, we should have, for example, two non-equivalent primitive binary groups (or two non-equivalent representations of the group) of order  $60\varphi$ ; namely the group ( $E$ ), page 73, and that group whose trans-

formations are obtained from the corresponding transformations of  $(E)$  by interchanging  $+\sqrt{5}$  and  $-\sqrt{5}$  (this cannot be brought about by a change of variables).

3. (BULLETIN, page 250.) That a primitive group in four variables cannot contain a transformation of order 9 follows from § 117, the reference to this fact being omitted by an oversight. By the reasoning of § 111 it is found that a tentative transformation of order 9 generates a Sylow subgroup of this order, and the presence in a primitive simple group of such a subgroup would exclude all the types given in § 111 except  $(a)$  and  $(g)$ . Since groups of order  $2^a \cdot 3^b$  are not simple, the primitive group in question could only be of the kind discussed in § 117.

4. (BULLETIN, page 250.) Concerning the statement "In view of . . . appears superfluous," the author remarks that as yet (§§ 106–108) it had not been shown that  $T, C, D, V, F$  generate a *finite* group; i. e.,  $K'$ , containing  $K$  as a subgroup, might as yet prove not to be of finite order. Hence the necessity for showing that  $K'$  and  $K$  are identical, and this is in part the purpose of § 109 (cf. §§ 115, 116, where the same problem is encountered for other groups).

5. The author hereby takes opportunity to emphasize that throughout the chapter on group characteristics, the term "simple group" has its customary meaning; i. e., a group which contains similarity transformations is not "simple" here, though it be given that the corresponding collineation group is simple. (Cf. § 51, 1°, concerning the use of the term in the chapters on groups in 2–4 variables.)

6. In conclusion the author submits the following additional list of errata (the first two were suggested by Professor F. N. Cole):

- Page 18, line 1, and page 55, line 16. Read "any set" instead of "any one set."
- " 129, " 16. Corollary 2 should read "Let . . . characteristics of two simply isomorphic groups . . ." instead of "Let . . . characteristics of two groups. . . ."
- " 70, " 14. The equation  $\alpha^g = 1$  should be  $\alpha^{2^g} = 1$ .
- " 73, " 6 above § 59. After  $T'^2 = E_1$ , insert:  $S'T' =$  a transformation of order  $3\varphi$ .
- " 104, " 5 from bottom. Replace the first  $x_2'$  by  $x_1'$ .
- " 121, last line. Replace  $(x_1) = \alpha x_1$  by  $(x_1)M = \alpha x_1$ .
- " 126, line 7 from bottom. Replace  $f$  by  $cf$ .
- " 127, " 8. The letters  $\bar{X}_1, \dots, \bar{X}_n$  should be accented:  $\bar{X}'_1, \dots, \bar{X}'_n$ .

Page 133, in Exercise 1, after “. . . consists of” insert “the group represented by  $E$  repeated 60 times.”

“ 133, footnote. Replace  $6^\circ$  by  $7^\circ$ .”

“ 182, line 20. The second of the set of numbers given, namely 6, should be 3.

“ 182. At end of footnote marked \* write: Jordan’s method is to express suitable powers of the solutions of the linear differential equation as rational functions of  $x$  and of all the roots of a certain algebraic equation. The degree of this equation will then generally be lower than the corresponding degree found above. See *Crelle*, vol. 84, pp. 93, 112.

H. F. BLICHFELDT.

### REMARKS ON ELLIPTIC INTEGRALS.

It is known that an elliptic integral of the first kind is everywhere one-valued, finite, and continuous on its associated Riemann surface, while the elliptic integral of the second kind is algebraically infinite, and the elliptic integral of the third kind is logarithmically infinite at certain points of the surface. This is a characteristic distinction of these integrals and is essential in their study. It is also true of the hyperelliptic and abelian integrals.

The Legendre form of the integral of the first kind is

$$F(k, \varphi) = \int_0^\varphi \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}}.$$

When  $k = 1$ , this integral becomes

$$F(1, \varphi) = \int_0^\varphi \frac{d\varphi}{\cos \varphi} = \log \tan \left( \frac{\pi}{4} + \frac{\varphi}{2} \right).$$

If further  $\varphi = \frac{1}{2}\pi$ , it is seen that the complete elliptic integral

$$F_1(1) = F \left( 1, \frac{\pi}{2} \right)$$

is logarithmically infinite, while  $F_1(0) = \pi/2$ .

As this is the only possible chance, remote though it be, for an integral of the first kind “to claim kin” with one of the third kind, I don’t see why a gentleman from Alabama, where relationships are cherished, the connection often being even more remote, should suffer a “jolt” (see the *BULLETIN*, Febru-