THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE thirty-first regular meeting of the San Francisco Section was held at Stanford University on Saturday, April 6. There were present eighteen persons, including the following twelve members of the Society:

Professor R. E. Allardice, Professor Harry Bateman, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Dr. Pauline Sperry, Professor R. M. Winger.

Professor W. A. Manning occupied the chair. At the close of the session attending members were entertained at luncheon, and in the afternoon an interesting demonstration of experiments on aeroplane propellers was given by Professors Moreno and Lesley, of Stanford University.

It was provisionally decided that the next meeting at Stanford University should be held on April 5, 1919.

The following papers were read at this meeting:

(1) Professor D. N. LEHMER: "On Jacobi's extension of the continued fraction algorithm" (preliminary report).

(2) Professor H. F. BLICHFELDT: "A second principle in the geometry of numbers."

(3) Professor M. W. HASKELL: "Continuous groups of quadratic transformations."

(4) Professor D. N. LEHMER: "Arithmetical theory of certain Hurwitzian continued fractions."

(5) Professor E. T. BELL: "Numerical functions of $[x]$."

(6) Professor E. T. BELL: "Fourier series for certain elliptic functions."

(7) Professor R. M. WINGER: "On the satellite line of the cubic."

(8) Professor R. M. WINGER: "Self-projective rational septimics."

Professor Bell's papers were read by title. Abstracts of the papers follow below.

1. Professor Lehmer developed new connections between

the ordinary continued fraction and the extension made by Jacobi in 1868, with applications to the theory of numbers and to the theory of equations.

2. In a former paper (*Transactions of the American Mathematical Society*, 1914, pages 227–235) Professor Blichfeldt proved a geometrical theorem having a number of applications in the theory of numbers; for instance, the variables of a positive-definite quadratic form f in n variables and of determinant D can be given such integral values, not all zero, that the numerical value of f is not greater than a number which has $(n/\pi e)D^{1/n}$ for its asymptotic value. This limit being too high, the question arises what may the actual lowest limit be; i. e., there is need of a theorem giving a limit which is exceeded by at least *one* quadratic form. Minkowski has given such a theorem in geometrical form (Collected Works, volume 2, page 270) without proof, namely: an oval surface (like an ellipse) in space of n dimensions, having a center at the origin and a volume $\leqq 2(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots)$, can be deformed by a homogeneous linear transformation which preserves volumes, into a surface which contains no lattice points except the origin. Professor Blichfeldt states a theorem which gives somewhat better results than that of Minkowski; thus, applied to ellipsoids, this theorem is that the volume that may be assumed approaches a number $\geqq 23$ for large values of n (instead of 2 by Minkowski's theorem).

3. Professor Haskell has examined the various types of continuous transformations of the plane which in homogeneous form are rational quadratic transformations. They are all six-parameter groups of mixed quadratic and linear transformations, with two-parameter subgroups which are purely quadratic. Besides those which are simple products of linear transformations of the separate variables, they may be classified as elliptic, hyperbolic, and parabolic—the elliptic type including as a special case the well known inversion group.

4. If A_m/B_m is the mth convergent of the Hurwitzian continued fraction

$$\overline{(a_1, a_2, a_3, \cdots, a_{k-1}, ma_k)} \qquad (m = 1, 2, 3, \cdots)$$

and A_m'/B_m' that of

$$(\overline{a_{k-1}, a_{k-2}, \cdots, a_2, a_1, -ma_k - 2M}), \quad (m = 1, 2, 3, \cdots)$$

and A_m''/B_m'' the $(m+1)$st convergent of

$$(x, a_{k-1}, a_{k-2}, \cdots, a_2, a_1, y, \overline{a_{k-1}, a_{k-2}, \cdots, a_2, a_1, -ma_k - 2M}), \quad (m = 2, 3, \cdots)$$

where

$$M = \frac{B_{k-1} + A_{k-2}}{A_{k-1}}, \quad x = \frac{B_{k-1} - A_{k-2}}{A_{k-1}},$$

$$y = -A_k - 2\left(\frac{B_{k-1}^2 + B_{k-2}A_{k-1}}{A_{k-1}B_{k-1}}\right)$$

and where $a_1, a_2, a_3, \cdots, a_{k-1}$ are positive or negative integers or zero, and a_k, A_{k-1}, B_{k-1} are positive or negative integers not zero, then Professor Lehmer shows

$$\begin{aligned} A_{\rho k-1} &= (-1)^{\rho-1}A'_{\rho k-1}, \\ A_{\rho k} &= (-1)^{\rho}(A'_{\rho k} + MA'_{\rho k-1}), \\ B_{\rho k-1} &= (-1)^{\rho-1}A''_{\rho k-1}, \\ B_{\rho k} &= (-1)^{\rho}(A''_{\rho k} + MA''_{\rho k-1}). \end{aligned}$$

From these equations he develops the arithmetical theory of such continued fractions, and shows, among other things, that for n prime to a_k, A_{k-1}, B_{k-1}, we have always

$$\begin{aligned} A_{2nk-1} &\equiv B_{2nk} \equiv 0 \pmod{n}, \\ A_{2nk} &\equiv B_{2nk-1} \equiv (-1)^{kn-1} \pmod{n}. \end{aligned}$$

In particular the denominators of the convergents of orders $3n$, $3n-2$, and $3n-6$, and the numerator of that of order $3n-3$. for the regular continued fraction which represents the base of Naperian logarithms are always divisible by n.

5. Simple methods are given by Professor Bell for finding relations between, and reducing, sums of numerical functions of $[x]$, the greatest positive integer contained in x. Such reductions are useful in the investigation of arithmetical mean values, etc. The paper appeared in part in the March number of the *Annals of Mathematics*.

6. In Professor Bell's second paper the series are for the squares and certain products of the complete set of eighteen doubly periodic theta quotients considered by Hermite in his memoir on Kronecker's class-number relations. The series have important arithmetical consequences. The paper has appeared in the *Messenger of Mathematics.*

7. While Salmon discusses the satellite line of the cubic at some length he fails to give its equation. The symbolical expression for this important covariant has been supplied by Morley. In the present paper Professor Winger derives the explicit equation of the satellite, both for the rational and general cubic, in canonical forms, and discusses associated loci. Several chain theorems are obtained and a generalization is made for the plane curve of order n.

8. Professor Winger's second paper, a preliminary report of which was made to the Society at a former meeting, is intended as a sequel to two papers of similar titles which have already appeared in the *American Journal of Mathematics.* The varieties of self-projective rational septimics in parametric form are exhibited and the more striking properties of certain of the curves are pointed out. According to the criterion of classification adopted, there are 26 distinct types, invariant under cyclic groups of order 2, 3, 4, 5, 6, and 7, dihedral groups of order 6, 10, and 14, and an infinite group.

B. A. BERNSTEIN,
Secretary of the Section.

THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY AT CHICAGO.

THE tenth regular meeting of the American Mathematical Society at Chicago, being also the forty-first regular meeting of the Chicago Section, was held on Friday and Saturday, April 12 and 13, at the University of Chicago. The various sessions were attended by about fifty persons, among whom were the following thirty-five members of the Society:

Professor R. P. Baker, Professor G. A. Bliss, Professor J. W. Bradshaw, Professor R. D. Carmichael, Professor A. R.