It seems almost superfluous to refer to the style of presentation of the results in this treatise. Every one in the least familiar with Darboux's writings is conscious of their lucidity and charm of style. And here we have the final product of this great expositor.

L. P. EISENHART.

SHORTER NOTICES.

On Growth and Form. By D'ARCY W. THOMPSON. Cambridge, University Press, 1917. 8vo. 16+793 pages. Price 21 shillings.

UNTIL his recent appointment to the chair of natural history at the University of St. Andrews, the author of this work was professor of natural history at University College, Dundee. He has long been prominent as a member of various fishery boards and conferences, serving, for example, as the British delegate to the Bering Sea fishery conference of 1897. He is the author of "A Glossary of Greek Birds," has edited Aristotle's Historia Animalium and various blue-books on North Sea investigations, and he has translated and edited Hermann Müller's Fertility of Flowers.

And now we have this elaborate volume containing the results of many years of observation and study in widely separated fields and constituting an "easy introduction to the study of organic form." The author tells us: "It is not the biologist with an inkling of mathematics, but the skilled and learned mathematician who must ultimately deal with such problems as are merely sketched and adumbrated here. I pretend to no mathematical skill, but I have made what use I could of what tools I had; I have dealt with simple cases, and the mathematical methods which I have introduced are of the easiest and simplest kind."

After the Introduction we have chapters with the following titles: 2. On magnitude; 3. The rate of growth (106 pages); 4. On the internal form and structure of the cell; 5. The forms of cells; 6. A note on absorption; 7–8. The form of tissues or cell-aggregates; 9. On concretions, spicules, and spicular skeletons; 10. A parenthetic note on geodetics; 11. The

logarithmic spiral* (94 pages); 12. The spiral shells of the foraminifera; 13. The shapes of horns, and of teeth or tusks: with a note on torsion; 14. On leaf-arrangement or phyllotaxis; 15. On the shapes of eggs and of certain other hollow structures; 16. On form and mechanical efficiency; and 17. On the theory of transformations or the comparison of related forms.

The whole work is written with fluency, lightness of touch, and notably attractive phrasing fraught with a wealth of suggestion. Many historical notes and full bibliographies indicate the author's thorough familiarity with the literature of his subject. The mathematics involved is indeed elementary, although some of the discussion refers to results whose derivation must have required the use of the theory of least squares and of probabilities† (for example, in Chapter 3), while other parts involve such subjects as are summarized by O. Fischer in his Physiologische Mechanik‡ (for example in Chapters 2 and 16). In conclusion let us note a few of the vast number of special topics discussed in a most interesting manner.

Some of the illustrations of "hexagonal symmetry" in the seventh chapter are: (1) artificial cellular tissue formed by the diffusion in gelatine of drops of a solution of potassium ferrocyanide; (2) "hexagonal epithelium-cells of the pigment layer of the eye, external to the retina"; (3) optical sections of soap froth imprisoned between glass plates; and (4) the bee's cell. We are told that "the curious reader may consult Sir Thomas Browne's quaint and beautiful account in the Garden of Cyrus,§ of hexagonal (and also of quincuncial) symmetry in plants and animals which 'doth neatly declare how nature Geometrizeth, and observeth order in all things.'" In the seven-page account of the bee's cell Thompson sketches the

* In connection with this chapter it is noted that "a great number of spiral forms, both organic and artificial, are described and beautifully illustrated in Sir T. A. Cook's Curves of Life, 1914, and Spirals in Nature and Art, 1903."

† An excellent American work which gives a good idea of some kinds of mathematics employed in biology is C. B. Davenport's Statistical Methods with Special Reference to Biological Variation. Third edition. New York, 1914.

‡ Encyklopädie der mathematischen Wissenschaft, Band IV, 1, II, Heft 1, 1904. Not referred to by Thompson.

§ Published in 1658.

correct history* of discussions of its form (by Pappus, Maraldi, Réaumur, Koenig, Maclaurin, Lhuilier, Buffon, P. Huber and Darwin), and the derivation, by calculus, of its minimum property.

The mathematician will more than once take exception to statements made in the two paragraphs on pages 440–441 concerning "geodetic" curves on surfaces. These curves are introduced in connection with the discussion of spicules, helicoid formations as in the coil which stiffens the tracheal tubes of an insect (Chapter 10), etc.

Discussion of leaf-arrangement leads to the series of ratios: 2/3, 3/5, 5/8, 8/13, 13/21, 21/34, 34/55, 55/89, 89/144, which are simply convergents of the continued fraction $\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\cdots$, etc. Thompson refers, incorrectly, to this "celebrated series" as "commonly called the Fibonacci series." This designation would be more correctly applied to the series of numbers 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 (each term of which, after the second, is the sum of the two preceding) which comes up in the solution of a rabbit problem discussed by Leonardo of Pisa in his Liber Abbaci.† Fibonacci's series is sometimes called Lamé's series.‡ It was well known to Kepler, who discussed it in connection with flower forms,§ and it is closely connected with the division into golden section|| and with a number of mathematical investigations by Binet, Lamé, Lucas, Catalan, Schlegel, and Landau.¶

* Incorrect statements, from the time of Réaumur, have been made so often in this connection (e. g., Mach, Science of Mechanics, 1902, p. 453) that the corrections indicated by R. L. Ellis in his essay "On the Form of Bees' Cells" should be noted (Mathematical and Other Writings, Cambridge, 1863, p. 353).

† Scritti, tome 1, pp. 283–284.

‡ Thompson makes a slip in connecting this series with " Father Bernard Lami, a contemporary of Newton."

§ F. Ludwig, "Kepler über das Vorkommen der Fibonaccireihe im Pflanzenreich," *Bot. Centralbl.*, 1896, Bd. 68, p. 7.

|| Two books on this subject are: (1) A. Zeising, Neue Lehren von den Proportionen des menschlichen Körpers aus einem bisher unbekannt gebliebenen und die ganze Natur und Kunst durchdringenden Grundgesetze. Leipzig, 1854; (2) F. X. Pfeifer, Der goldene Schnitt und dessen Erscheinungsformen in Mathematik, Natur und Kunst. Augsburg, 1885 —but Thompson regards the numbers of golden section as "devoid of biological significance" (p. 649).

¶ Cf. *Amer. Math. Monthly*, May, 1918.

In *L'Intermédiaire des Mathématiciens* for 1895* Monsieur Henri Brocard suggested a study of the "fruit de l'hélianthe" —a comparison of the arrangement of the florets of the sunflower with such curves as spirals, epicycloids and the involute of a circle. In 1909 a remarkable sunflower photograph taken by M. Brocard was published along with notes: (1) by J. W. N. Le Heux, who found "lignes rapellant tout à fait la spirale d'Archimède (ou la spirale logarithmique), ou même d'autres spirales, selon la loi de variation du rayon vecteur," and (2) by E. Maillet who wrote as follows:

"Je ne puis m'empêcher d'être frappé de l'analogie de disposition que présentent les spirales de la photographie de M. Brocard avec celles qu'a rencontrées M. le commandant Hartmann, dans ses études sur la déformation des métaux, pour le cas d'un cylindre creux soumis à une pression intérieure, ou extérieure, ou pour le cas d'une plaque poinçonnée en son centre [*Revue d'Artillerie*, novembre (page 99–100) et décembre 1894, 1895 et 1896].

"M. Mesnager a montré (*Revue d'Artillerie*, mars 1898) qu'on pouvait, au moins en partie, expliquer la production de ces lignes en admettant que, dans la déformation d'un corp donné soumis à des efforts: 1° le glissement tend à se produire en un point le long de l'élément plan suivant lequel la valeur absolue du rapport R de l'effort tangentiel à l'effort normal (cohésion comprise) est maximum; 2° ce glissement, qui constitue *une déformation permanente*, se produit effectivement dès que R atteint une valeur constante f (analogue à un coefficient de frottement) caractéristique du métal.

"Voir encore Appell, *Bull. Soc. Math.*, tome XXVIII, 1900, p. 66.

"Je ne suis pas assez botaniste pour dire si des phénomènes de déformation tout à fait analogues au point de vue de la Mécanique peuvent se produire dans le développement de la

* Tome 2, p. 134, question 524; reproposed 1902, tome 9, pp. 138–139. Notes: 1904, tome 11, p. 95; 1908, tome 15, pp. 173–174; 1909, tome 16, pp. 101–102.

In response to another question in *L'Intermédiaire* (1688, by Espanet, Dec. 1899, tome 6, p. 269), concerning surfaces of sea shells, more particularly those of spiral form, Brocard, Godefroy and Escott gave a number of references to the literature of the subject (1900, tome 7, p. 40; 1901, tome 8, pp. 167, 314; 1910, tome 17, p. 155), nearly all of which, and many more, seem to have been considered by Thompson. One notable work which he appears to have overlooked is Haton de la Goupillière's "Surfaces Nautiloïdes" which occupies almost the whole of the third volume of *Annaes scientificos da Academia polytechnica do Porto*, Coimbra, 1908.

fleur et du fruit de l'hélianthe (ou d'autres plantes) et si cela peut influer sur la disposition de la mosaïque ou même la déterminer."

According to the volume before us* it seems that there is no doubt about the logarithmic spiral arrangement in the sunflower, or indeed that we have systems intersecting isogonally. Reference is made to a careful and elaborate study by H. H. Church,† which has been adopted by T. A. Cook in his Curves of Life. But of Thompson's comment in this connection I shall not do more than quote a single sentence: "On the analogy of the hydrodynamic lines of force in certain vortex movements, and of similar lines of force in certain magnetic phenomena, Mr. Church proceeds to argue that the energies of life follow lines comparable to those of electric energy, and that the logarithmic spirals of the sunflower are, so to speak, lines of equipotential."

Professor Thompson's work is recommended as one of the most notable and most readable of scientific books appearing in the past decade.

R. C. ARCHIBALD.

Hvorledes Mathematiken I Tiden Fra Platon Til Euklid Blev Rationel Videnskab. By H. G. ZEUTHEN. Reprinted from the D. Kgl. Danske Vidensk. Selsk. Skrifter, Naturvidensk. og Mathem. Afd., 8. Række, I. 5. Copenhagen, 1917. 183 pp.

ANY work by Professor Zeuthen on the history of mathematics, even though it be a reprint from the memoirs of an academy, deserves to be brought to the attention of other scholars than those who may chance to see the original publication. This is especially true when the memoirs of the academy are printed in a language not generally familiar to scholars and therefore are not as frequently consulted as those which appear in languages more nearly international.

In this particular case there is the more reason for calling attention to the memoir because a summary is given in the French language so that all scholars may have easy access to the argument and, with this as a guide and with a knowledge of German, may follow the more important details in the text itself.

* Pp. 639–640.

† Relation of Phyllotaxis to Mechanical Laws, Oxford, 1901–1903; cf., *Ann. of Botany*, vol. 15 (1901), p. 481.