two symbols of integration operate upon the same class of functions and give the same integral value for the same function. The other is in reality a pseudo-equivalence expressed in the fact that an integral of one kind may be transformed into an integral of another kind, the functions integrated in the two cases being different. A careful and instructive analysis of these equivalences is given by Hildebrandt.

To one result of this analysis it is desirable to have attention sharply directed. The Stieltjes integral seems destined to play in the future a rôle of central importance in processes of integration and summation. The Lebesgue integral when introduced received almost immediate attention and recognition and found its way rapidly into the main current of mathematical thought; but the Stieltjes integral has been singularly neglected notwithstanding its inherent simplicity and naturalness. Through the summary of its properties given by Hildebrandt and the applications mentioned one is convinced of its central importance and is led to expect it to assume a new place in mathematical thought. In this connection it is of particular interest to note also Hildebrandt's extension of the Stieltjes integral modelled on the Lebesgue extension of the Riemann integral.

We conclude with the following list of misprints in de la Vallée Poussin's monograph: page 20, last line, write $m(F_1 + F_2) = mF_1 + mF_2$; page 28, second theorem, write $(f \geqq A)$ instead of $(f \geqq a)$; page 34, line 16, write $\omega 2$ instead of ω_2; page 96, end of second paragraph, write Df instead of DF; page 126, line 9 below, write "Une" instead of "Uue"; page 133, line 2, write "de classe $\leqq \alpha$."

R. D. CARMICHAEL.

SHORTER NOTICES.

Leçons de Mathématiques Générales. Par L. ZORETTI. Paris, Gauthier-Villars, 1914. 8vo. xvi + 753 pp.

Exercices numériques de Mathématiques. Par L. ZORETTI. Paris, Gauthier-Villars, 1914. 8vo. xv + 125 pp.

THE author of this text and its accompanying set of exercises is well qualified for the task. Formerly instructor in the lycée, contributor to the theory of functions by several important memoirs, professor of mechanics, he is fully equipped to add to several already existing treatises on mathematics in general another of the particularly successful French treatises of this kind. His special object was to write a book that would furnish the necessary knowledge for the most widely different classes of students, whether they intended to enter experimental science, engineering, the ranks of teaching, or some of the professions. His goal is that of applications, and everything is included that might be useful for practical mathematics, everything is excluded that cannot at once justify its applicability elsewhere.

This program, to be realized in a manageable space, requires a presentation which is as intuitive as possible, but still attacking with all frankness demonstrations which must be given, and further considering carefully the conditions under which the results that are arrived at may be applied, and the significance of these results. Exactness in definition is also necessary in order that the student may clearly understand what he is doing. The student is thrown upon his own resources, particularly in the exercises, many of which demand careful construction and measurement by the student himself. We might say that a certain reasonable amount of laboratory mathematics is insisted upon in order that the student may learn not only to apply mathematics to data given in some way, but to derive the data to which the mathematics is to be applied.

The first 157 pages are devoted to a survey of analytical geometry, including the theory of vectors. This grouping of the subjects is perfectly proper inasmuch as analytical geometry is merely a rather primitively developed vector analysis. The first chapter defines vector (directed segment), the equivalence of vectors which are on the same axis, and the equipollence of vectors which are on parallel axes. Couple is included, as well as opposed vectors. Addition and subtraction of vectors on the same axis is defined, as well as the same processes for vectors whose axes are concurrent. These notions are extended to directed arcs and angles. The theorems of projection follow naturally.

The next chapter defines and illustrates the different kinds of coordinates for surfaces and for space. The method of the book is very well shown in the way in which the systems of curvilinear coordinates are defined and their uses indicated. This kind of coordinate is seldom mentioned in the beginning of the usual texts. The elementary formulas for a line are developed, and the determination of mean distances for n points is introduced as an example, thus putting the student in touch at once with possible uses for his knowledge. The coordinates of a vector are considered next, followed by the determination of any figure, bringing in at the same time Euler's angles, and degrees of freedom. The determination of lines on surfaces, and surfaces themselves follow. Then the methods of changing the system of coordinates are developed. These ideas are then to be applied to such exercises as the following: Choose proper parameters for the following figures: segment of a line located in a given plane; circle in space; given cone of revolution, the vertex in a given plane; circle tangent to a given line in space; coordinates of a point on a given line or surface.

It seems to the reviewer that such a method as this marks a distinct advance over the usual one of explaining the rectangular and polar coordinates followed by a few perfunctory plottings, and then a development of formulas for the line and plane. The student is generally supposed to seize instantly the idea of coordinates and their significance, and to be able to reason by their means with no further study of them. Their significance as parameters or numbers which may arise in an infinity of ways, different sets being useful for different purposes, is seldom pointed out, and the student is generally unable to state his problem in any other way than by the conventional methods of x, y, z or r, θ, φ. When he comes to integration in space he is usually helpless.

Chapter three develops the fundamental formulas of the line and plane. Chapter four considers the theory of vectors, moments, resultants, resultant moments, equivalent systems, invariants, central axis, couples, coplanar vectors, and parallel vectors. The student is early equipped with useful knowledge for statics. Two chapters follow on circles and conics. In the latter the approximate construction by circular arcs is described. In the manual of exercises the second series contains other methods of drawing these curves, with some original theorems on them easily deduced from the methods. Chapters seven and eight consider quadrics and some of the common curves and surfaces.

The second part of the work is given up to algebra, theory of functions, and derivatives. It occupies 342 pages, the greater part being concerned with functions. The algebraic topics cover complex numbers and their representation, the binomial theorem, determinants, and elimination. A chapter is devoted to infinite series, with the convergence tests of D'Alembert, Cauchy, and the rule that is derived by comparison of the series with the series Σn^{-p} where $1 < p$. These are characterized for practical purposes as follows: The last reveals only slowly convergent series, the second is not easy of application, so that the rule of D'Alembert is about the only practical one.

The notion of function is based upon dependence, but the illustrations are of natural dependence and not of the artificial type often seen in such examples. The author then defines, and considers the uses of, entire functions, simply periodic functions (a vastly better name than harmonic functions), the exponential function, circular functions as exponential, hyperbolic functions, logarithmic function, inverse circular functions, inverse hyperbolic functions, with some closing remarks on graphs when the quantities represented by x and y are not of the same kind.

In discussing differentiation examples are freely used to develop some very fundamental notions, often not mentioned in the beginning works on calculus studied by engineers and other practical students. For instance, it is shown by a wave curve that lies in a very narrow strip, that while functions which have derivatives which lie close together do not differ much themselves, on the other hand functions which lie close together may have derivatives very far apart. For this reason integration never has the hard conditions to meet which restrict derivation. Another instance of departure in a useful way from other elementary presentations is in the definition and use of the directed derivative of scalar and vector point functions, notions much more useful than a large set of complicated algebraic formulas for differentiation. A chapter of variations of functions presents several well known curves and some of their peculiarities. A short chapter on development in series contains the essentials. Derivatives are applied to curves, surfaces, and movements, including relative motion. A chapter is given to theory of equations and their solutions, and a chapter to numerical and graphical calculation.

The third part of the work covers 248 pages, and considers the integral calculus and its applications. The integral is defined as the limit of a sum, making the definite integral properly the foundation of the whole subject. The connection with the primitive of the simple differential equation $dy = f(x)dx$ is immediately brought out, and some of the formulas of differentiation, as $d \cdot uv = vdu + udv$, are interpreted in integration, this one leading to integration by parts. The calculation of mean values is given as an example of the use of the definition. A chapter follows on general methods of finding indefinite integrals. Chapter three extends the notion in some directions, as that of improper integrals, line integrals, double and triple integrals, leading up to the formulas of Green and Ostrogradsky, although singularly the complete formula of Stokes is omitted at the point where it would have naturally come in. A chapter is given to elliptic functions, one to Fourier series, one to geometric applications, one to applications to mechanics. In the latter the notions of vector fields, curl, divergence, flux, vector lines and tubes, work, circulation, level surfaces, and potential are brought in. The approximate calculation of integrals has a chapter. The last two chapters consider differential equations, total and partial, and though quite elementary, the author has nevertheless given the student a good working basis. The accompanying problems of the manual of exercises reenforce the text materially.

The author states that he considers the needs of the student who has to study by himself, with little or no assistance. This no doubt accounts for the very plain treatment, and would suggest that in general a text written for students will be clearer than one written for the use of teachers. Of course a certain amount of rigor in books written for teachers is demanded, but rigor does not always lead to usable knowledge. Professor Zoretti is to be congratulated on his success.

JAMES BYRNIE SHAW.

College Algebra with Applications. By E. J. WILCZYNSKI. Edited by H. E. SLAUGHT. Boston, Allyn and Bacon, 1916. xx+507 pp.

THIS algebra, unlike the traditional college algebra, possesses unity, the centralizing theme being the function concept. The book opens with an excellent chapter on the number sys-