from which a very interesting connection with Severi's theory of the base is derived. The paper appeared in the *Rendiconti dei Lincei* for February.

ARNOLD DRESDEN,
Secretary of the Section.

THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-ninth regular meeting of the San Francisco Section was held at Stanford University on Saturday, April 7. Two sessions were required for the presentation of the program. Professor Lehmer occupied the chair. The following members of the Society were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Professor E. W. Ponzer, Dr. H. N. Wright.

It was voted to hold the next meeting of the Section at the University of California, on October 27.

The following papers were read at this meeting:

(1) Professor H. F. BLICHFELDT: "A further reduction of the known maximum limit to the least value of quadratic forms."

(2) Professor D. N. LEHMER: "Certain divisibility theorems concerning the convergents of Hurwitzian continued fractions."

(3) Dr. G. F. MCEWEN: "Determination of the functional relation between one variable and each of a number of correlated variables by successive approximation."

(4) Professor W. A. MANNING: "On the order of primitive groups (III)."

(5) Dr. H. N. WRIGHT: "Note on a certain quadratic transformation of the plane."

In the absence of Dr. McEwen, his paper was read by Mr. E. F. Michael of the Scripps Institute for Biological Research. Abstracts of the papers follow below.

1. Having given the determinant D of a positive-definite quadratic form F in n variables, such integers, not all zero,

can be assigned to the variables that the value of F is not greater than $\gamma_n D^{1/n}$, where γ_n is a number depending only upon n (cf. *Transactions* for 1914, pages 227 ff.). Professor Blichfeldt has recently obtained a lower value for γ_n than that hitherto known (l. c., page 233) when $n > 5$.

2. Hurwitz (*Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, 1896) has studied continued fractions whose partial quotients belong to arithmetical series. Taken modulo n the partial quotients of such fractions recur, and so also do the convergents. The number of terms in the period of the convergents seems to depend in a very simple way on the modulus, and the appearance of prime factors in the numerators and denominators of the convergents presents an interesting study. The continued fraction for the Naperian base $e = (2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, \cdots)$ is an example. Calling the kth convergent A_k/B_k, Professor Lehmer has shown that A_k recurs with a period of $3n$ with respect to any modulus n, and B_k has a period $6n$. Also $B_{3n-2} \equiv B_{3n-6} \equiv A_{3n-3} \equiv 0 \pmod{n}$.

3. In order to determine quantitative relations from "field" data, where all the related magnitudes vary simultaneously, the disturbing effect due to the interrelationship of the independent variables must be eliminated. Assuming the change in the dependent variable w, due to a given change in any one of the independent variables x, y, z, etc., to be independent of the values of the other variables, the problem is to determine the functional relation of w to each variable, when the form of the function is unknown and unrestricted.

Arrange the values of x in order of magnitude, and enter corresponding values of w. Divide this series of values of x into groups, each containing about the same number of entries and having about the same range, and find the regression of w on x in each group, assuming it to be linear in each. Proceed in the same way with each of the other independent variables, thus obtaining first approximations to the average value of w and to each regression coefficient in each group corresponding to the average value of the independent variable from which the group was formed. By means of these approximate relations of w to y, w to z, etc., apply a correction to each entry in the (w, x) tabulation that will reduce the value of w

to what it would have approximately been if y, z, etc., had constant values, arbitrarily chosen. Then redetermine the regression of w on x in each group, and proceed in the same way with the other independent variables.

A set of determinate simultaneous linear equations can be derived whose roots are the true regression coefficients and averages required, and it can be shown that if the values of the averages and regression coefficients found by continuing the above approximation process converge, the limits will satisfy these equations.

Formulas, readily derived and involving constants easily determined from the data, greatly facilitate the computation, and provide an absolute check on the numerical work of each approximation, after the second.

Dr. McEwen will offer this paper to the *Annals of Mathematics*.

4. A partial statement of Professor Manning's theorem is this: Let q be any positive integer greater than unity, and let p be a prime number greater than $2q$; then the degree of a primitive group that contains a substitution of order p and degree qp but none of order p and of degree less than qp cannot exceed $qp + 4q - 4$. This paper has been offered to the *Transactions*.

5. An involution of lines is set up about each of the points A and B such that the double lines of each involution are at right angles. Then any point P of the plane, as the intersection of a line of A with a line of B, corresponds to a point P' as the intersection of the corresponding lines of A and B. And conversely P' is seen to correspond to P. In this way is obtained a quadratic transformation of the plane.

The involutions about A and B determine a third involution of the same character about a point C, such that the same quadratic transformation of the plane is obtained from any two of the three involutions.

Dr. Wright shows that in this transformation the ideal line of the plane goes into the circle through A, B, and C. This circle is found to be the nine-point circle of any triangle whose vertices are three of the four invariant points of the transformation.

W. A. MANNING,
Secretary of the Section.