

SHORTER NOTICES.

The Human Worth of Rigorous Thinking. By CASSIUS J. KEYSER. Columbia University Press, 1916. 8vo, vii+314 pp. \$1.75.

THESE essays and addresses of Professor Keyser are collected together in one volume as the result of fifteen years' consideration of various phases of mathematics, particularly of its significance as a branch of human knowledge. Everyone who has heard or read the different articles already knows that the author sees things with a poet's eye, and hears the strains of subtle music that emanate from this highest ether of the reason. The closing sentence of his book, and of the lecture widely delivered on mathematics, finds the deep-centered source of not only mathematics but also of science in the "joyous mood of the eternal Being" and both are considered to be a "sublimated form of play, the austere and lofty analogue of the kitten playing with the entangled skein or of the eaglet sporting with the mountain winds." For him "mathematics is but the ideal to which all thinking, by an inevitable process and law of the human spirit, constantly aspires." Consequently to challenge the value of mathematics as a study worthy of humanity is to challenge the worth of all thinking. Its significance to man is beyond measure, for "Transcending the flux of the sensuous universe, there exists a stable world of pure thought, a divinely ordered world of ideas, accessible to man, free from the mad dance of time, infinite and eternal." Its teaching may be so humanized that it will become a delight and a pleasure to all students, for "mathematics is precisely the ideal handling of the problems of life, and the central ideas of the science, the great concepts about which its stately doctrines have been built up, are precisely the chief ideas with which life must always deal, and which, as it tumbles and rolls about them through time and space, give it its interests and problems, and its order and rationality." The first three and the last two addresses of the collection are devoted primarily to considerations of the sort we have just mentioned.

There are four essays dealing with such fundamentals of thought as the figure and dimensions of the universe, hyper-space, infinity, and the existence of the hypercosmic. Two

others are closely related to these in that they discuss the logical reduction of mathematics and the multiple interpretations of systems of postulates and the deductions from them. In these we find the same lofty strain insisting upon the ability of the reason to find ultimate knowledge in mathematical theorems. "It is not in the world of sense, however precious it is and ineffably wonderful and beautiful, nor yet in the still finer and ampler world of imagination, but it is in the world of conception and thought that the human intellect attains its appropriate freedom—a freedom without any limitation save the necessity of being consistent." He is audacious enough to foresee an increase in our imagining powers so that even the ability to picture n -dimensional spaces may some day be gained by the race. In the hypercosmic he finds a world of pure thought "where is every type of order and manner of correlation and variety of relationship," rich in æsthetic interests, furnishing a sublimed and supersensuous art, the sure support of religious aspiration, permanent through all time. In hyperspaces he finds entities that have precisely whatever kind of existence may be attributed to the space of ordinary geometry. Whether such entities are found by the intellect or are created by it makes little difference.

There are also four essays on education, discussing the basis of a liberal education, graduate instruction, the functions of a university, and research in American universities. He takes issue with the doctrine that all subjects are equally valuable in a curriculum, pointing out that there are deep-rooted instincts in mankind which seek for certain things and for which any curriculum must provide. The study of the history of human activity and thought is a necessity, as well as the study of natural science; the study of mathematics, logic, or rigorous thinking in some form is not to be omitted, nor is that of the social character of man; but in a way surmounting them all is the study of beauty in all its forms, for without this, life would indeed be poor and depressing. Although the various topics that will respond to these needs may vary from century to century yet in some form or other they must always be present if education is really to be liberal in the highest sense. Would that everyone could appreciate the deep underlying principles set forth here, in this day of educational fads and crude radicalism! A university he finds is "the offspring and the appointed agent of the spirit of

inquiry, expression and servant of that imperious curiosity which in a measure impels all men and women, but with an urgency like destiny literally *drives* men and women of genius, to seek to know and to teach to their fellows whatsoever is worthy in all that has been discovered or thought, spoken and done in the world, and at the same time seeks to extend the empire of understanding endlessly in all directions throughout the infinite domain of the uncharted and unknown." As to research, the author believes in the three-fold organization of a university staff, the administration, the teaching staff, and the research staff. He characterizes his conceptions of the three thus: the great administrator is a man of constructive genius, the great teacher is a source of inspiration, the great investigator is a discloser of the harmonies and invariance hid beneath the surface of seeming disorder and of ceaseless change.

The delightful style of the author, permeated as it is with lofty sentiment, keen appreciation of beauty, wealth of imagery, striking illustrations, wide choice of terms, classical allusion, and warm feeling, makes the reading of these essays on rather abstract philosophical topics a pleasure as well as a profit. Every student of mathematics should read the book and catch the inspiration of enthusiasm for the divine science. "Mathematics is, in many ways, the most precious response that the human spirit has made to the call of the infinite and eternal. It is man's best revelation of the 'Deep Base of the World.'"

JAMES BYRNIE SHAW.

Four Lectures on Mathematics. (Delivered at Columbia University in 1911.) By J. HADAMARD, Member of the Institute, Professor in the Collège de France and in the Ecole Polytechnique, Lecturer in Mathematics and Mathematical Physics in Columbia University for 1911. New York, Columbia University Press, 1915. v + 52 pp.

ALL those who have had the pleasure of hearing Professor Hadamard's lectures have doubtless remarked his unusual facility in opening up wide vistas in the course of a relatively brief discussion. It is natural to expect such a facility to appear at greatest advantage in a set of lectures that are intended to be primarily inspirational, such as the above, and in this case the expectation is amply realized. By the omission of practically all technical details, Professor Hadamard has