SHORTER NOTICES.

The Teaching of Algebra (including Trigonometry). By T. PERCY NUNN, M.A., D.Sc. London, Longmans, Green and Company, 1914. xvi + 616 pp. Price $2.

Exercises in Algebra (including Trigonometry). By T. PERCY NUNN. Part I. 1913. xii+422 pp. Price $1.10. Part II. 1914. xii+552 pp. Price $1.75.

A FEW years ago one would have been justified in expressing surprise had an American teacher of mathematics gone to England to study modern tendencies in secondary education. To-day, however, if one were to study the present efforts at reform in the teaching of mathematics, he would certainly go to England for some of the most prominent leaders in the movement. Fifteen years have wrought a great change, and while the change has not been along the exact lines suggested by Professor Perry there is no doubt that he was the most influential of those who awakened the teachers of mathematics in the English-speaking world from the lethargy into which they had fallen. Naturally, the movement to better the work in mathematics has not been uniformly successful; for many ill-considered attempts have been made in addition to the few which give reasonable promise of producing satisfactory results. England has had her extremists as America has had hers, but on the whole she has been more consistent than we in her insistence upon a high degree of scholarship.

Among the strongest advocates both for reform in teaching and for a maintenance of high scholarship is Dr. Nunn, vice-principal of the London Day Training College, a teacher of experience and a man who adds character to the title of educator. In the publication of the three works under review, Dr. Nunn has doubtless been assisted by the judgment of the editors of the series of which these books are a part, Messrs. Abbott and Jackson and Dr. Macaulay, three of the best-known leaders in the progress of secondary mathematics in England.

The general introduction considers the nature of algebra, the formula, the graph, and the questions of method and the curriculum. Part I first discusses non-directed numbers, then directed numbers, and finally logarithms. Part II is concerned chiefly with the trigonometry of the sphere (in

particular, projections), complex numbers, periodic functions, limits, and statistics. Each of these topics is fully amplified by the accompanying exercise manuals where abundant material is given for the student's mastery of the respective chapters.

To the American teacher it will be of chief interest, so far as a review is concerned, to have attention called to a few of those features of the books which reveal the new and richer field which it is proposed to open to the English student. One of the advantages of so doing is that we Americans may see a serious effort at improving algebra without making the subject so soft and insipid as to have no interest for any healthy student. If the reader shall conclude that the teaching of algebra, as here treated, means merely the teaching of Dr. Nunn's algebra, this will not militate against the fact that the book will be found suggestive and helpful to teachers who follow courses which are quite at variance with the one which the author here lays down.

In the first place the reader will be struck by the fact that Dr. Nunn is not merely a mathematician but a philosopher and a psychologist as well. A single excerpt will reveal this fact, but it may be added that the fact is commonly apparent as one reads the pages of the work. "Mathematical truths always have two sides or aspects. With the one they face and have contact with the world of outer realities lying in time and space. With the other they face and have relations with one another. . . . From its dim beginnings by the Euphrates and the Nile mathematics has been on the one hand a means by which man has constantly increased his understanding of his environment and his power of manipulating it, and on the other hand a body of pure ideas, slowly growing and consolidating into a noble rational structure. . . . Our purpose in teaching mathematics in school should be to enable the pupil to realize, at least in an elementary way, this twofold significance of mathematical progress. A person, to be really 'educated,' should have been taught the importance of mathematics as an instrument of material conquests and of social organization, and should be able to appreciate the value and significance of an ordered system of mathematical ideas. There is no need to add that mathematical instruction should also aim at 'disciplining his mind' or giving him 'mental training.' So far as the ideals intended by these phrases are

sound they are comprehended in the wider purpose already stated. . . . The theoretical questions which are of most importance in an elementary course are just those which arise naturally out of attempts to apply mathematical ideas and methods to practical purposes."

With this principle to guide him, Dr. Nunn proposes to consider those parts of algebra which have the highest practical value, and if at times he seems to depart from the path he lays out it should be said that this is with the definite intent to develop his theoretical discussions from a consideration of the practical. In America we have the pedantic term "motivation" to designate this method of approach to the theoretical, but it adds nothing to the idea.

The author's idea is the sound one that the point of approach to algebra is the formula, "illustrations being drawn largely from the 'Pocket Book' of the engineer and similar formularies. . . . A little later comes the study of those manipulations of a formula by which it may be made to yield truths unknown or unperceived before. . . . The gradual elaboration of the formula as an instrument of description and investigation is, then, the first business of the course in algebra."

To carry out this idea the author proposes a fusion plan which has distinct advantages, differing herein from the ill-considered ideas occasionally met of fusing such subjects as algebra and geometry. His idea is that trigonometry, being largely a science of formulas and identities, should grow out of algebra, and that the calculus should do the same. These ideas seem practical, although the details of their execution may vary under such conditions as our colleges impose upon the secondary schools. Carrying out his plan, the author not only introduces work in plane trigonometry, but proceeds to the trigonometry of the sphere, Mercator and great-circle sailing, and map projections. His study of complex numbers develops into a study of circular functions, this into periodic functions, this into progressive and stationary wave motion, and this into hyperbolic functions. The course ends with a very satisfactory presentation of limits, including the elements of differentiation and integration, and with a helpful introduction to the mathematics of statistics.

In the work in graphs Dr. Nunn takes a very sane stand. The extremes of a few years ago find no place in his plan. He recognizes the value of the graph, and gives it a large place

in his scheme, with many examples to show its use; but he also recognizes that "it is inferior to the symbolic formula in many important respects. Its accuracy depends largely upon mechanical or non-intellectual conditions, such as the skill of the draughtsman and the exactness of the squared paper. It is less compact and less easily reproduced. Its message is frequently inarticulate and obscure. For these and similar reasons it should be regarded as a subsidiary algebraic instrument which fulfills its best office when it either leads up to a formula by which it may itself be superseded, or serves to unfold more fully the implications of a formula whose properties have been only partially explored."

After such a carefully considered statement the reader will probably be surprised that Dr. Nunn should resort to graphic treatment in certain cases where it is, to say the least, of doubtful value. Such cases are his elaborate development of the identities $ac + bc = (a + b)c$, $ac - bc = (a - b)c$, and especially $a^2 - b^2 = (a + b)(a - b)$, and the colored graphs in connection with negative numbers.

Another feature in Dr. Nunn's scheme that will strike the reader as of very doubtful value is his postponement of the negative number. Under the older plan of teaching, this delay would have been desirable; but the facility with which the negative number is now illustrated, and the ease with which pupils grasp the idea, renders such delay unnecessary. If the theory is to be presented as scientifically as the author proposes, there is some reason for waiting, but it is probable that the loss exceeds the gain at this stage of algebra.

It is not desirable in a review of this nature to consider further the large number of details that are found in three volumes aggregating over 1,500 pages. The present review has already extended beyond the limits usually allowed in the BULLETIN, but it is proper to add that there has never before appeared a work so helpful to a teacher of algebra. The reviewer does not believe in the sequence that Dr. Nunn lays down; he does not believe in certain uses of the graph that are set forth; he does not believe in several of the efforts at "motivation" which are advanced; and he does not believe that we are ready, in this country at any rate, to undertake certain parts of the course which is planned. But on the other hand he has not failed to recommend the work to hundreds of teachers of mathematics as the most noteworthy

contribution to the teaching of algebra that has yet appeared. Teachers who know our American problem, who are well enough balanced not to be enticed into fields that are certain to resist cultivation at the present time in this country, and who are searching for sane methods of reform, should read the pages which Dr. Nunn has here written with such care and erudition, and with such force and clearness.

DAVID EUGENE SMITH.

Syllabus of Mathematics. A symposium compiled by the Committee on the Teaching of Mathematics to Students of Engineering. Published by the Society for the Promotion of Engineering Education, Ithaca, N. Y., 1912. 136 pp.

THE purpose of this syllabus is to collect those principles and methods of mathematics which should constitute the minimum mathematical equipment of the student of engineering, or "those things for which a student ought never to be obliged to refer to any book—the things which he should have constantly at his fingers' ends." The book contains separate syllabi on elementary algebra (14 pages), elementary geometry and mensuration (7 pages), plane trigonometry (19 pages), analytic geometry (28 pages), differential and integral calculus (44 pages), and complex quantities (3 pages) together with a report of the discussion of these syllabi at the Pittsburgh meeting of the Society (14 pages).

Such syllabi will be of great value to students or teachers who wish to review the essentials of elementary mathematics courses. They will especially aid those students who end courses without proper perspectives. However, some may see danger of low standards in so much stress upon the *minimum* mathematical equipment of an engineer. The ideal of the committee is hardly realized, for few engineers "know by heart" and never need to "look up in a book" all the material in these syllabi. While admitting the value of such synopses of minimum essentials, yet as pointed out in one discussion (page 126), there is also need for lists of *all* the topics and principles that should be included in mathematics courses for engineers since there is some danger of too little rather than too great mathematical equipment.

Only few illustrative problems appear. A valuable supplement would be two sets of problems; one set giving an illustrative problem corresponding to each principle; the