10. In Professor Moore's paper an example is given of a function continuous in an interval $0 \leqq x \leqq 1$, whose development in Bessel's functions is not summable $(C\lambda)$ at the point $x = 0$, for any value of λ in the interval $0 \leqq \lambda < \frac{1}{2}$.

II. In the report of the colloquium, pages 85–88, Professor Veblen's subject matter appears distributed in six "Lectures," whereas only five lectures were actually delivered by each author. The headings in Professor Veblen's synopsis represent certain divisions of the subject matter, not the division into lectures, and the word "Lecture" should have read "Section."

THE MAXIMUM NUMBER OF CUSPS OF AN ALGEBRAIC PLANE CURVE, AND ENUMERATION OF SELF-DUAL CURVES.

BY PROFESSOR M. W. HASKELL.

(Read before the San Francisco Section of the American Mathematical Society, October 24, 1914.)

IT is well known that the double points of a rational algebraic curve can not in general all be cusps, and the maximum number of cusps has been determined in certain special cases. It is not difficult to find the maximum number from the consideration that none of the numbers in Plücker's equations can be negative.

Let m be the order and n the class, d the number of double points, k of cusps, i of inflexions and t of double tangents. We may first assume $d = 0$. In this case

$$2t = [m^2 - 9 - 3k][m^2 - 2m - 3k]$$

and the following inequalities must be satisfied:

(1) $$3k < m(m - 1),$$

(2) $$8k \leqq 3m(m - 2),$$

(3) $$2k \leqq (m - 1)(m - 2),$$

(4) Either $3k \leqq m^2 - 9$ and $3k \leqq m^2 - 2m$ simultaneously or else

(5) $3k \geqq m^2 - 9$ and $3k \geqq m^2 - 2m$ simultaneously.

Now, if m is greater than 9, $m^2 - 9$ is greater than $m(m-1)$ while if $m = 7, 8, 9$, $m^2 - 9$ is greater than $\frac{9}{8}m(m-2)$, so that if m is greater than 6, (4) must be satisfied. Hence, if m is greater than 6,

$$k \leqq \tfrac{1}{3}m(m-2).$$

Indeed, this formula is satisfied for $m = 3$ and $m = 5$, as is well known. The only exceptions are therefore $m = 4$ and $m = 6$.

Now, if m is of the form $3k$ or $3k + 2$, it is easy to show that a curve with the maximum number of cusps can have no further double points. The curve is self-dual, and

$$n = m, \quad d = t = 0, \quad k = i = \tfrac{1}{3}m(m-2),$$

$$p = \tfrac{1}{6}(m-2)(m-3).$$

If, however, m is of the form $3k + 1$, the curve may have one double point in addition. This curve is also self-dual, and

$$n = m, \quad d = t = 1, \quad k = i = \tfrac{1}{3}[m(m-2) - 2],$$

$$p = \tfrac{1}{6}(m-1)(m-4).$$

This result allows us to enumerate all self-dual curves. For, if $n = m$,

$$k = i = m - 2 + 2p,$$

$$d = t = \tfrac{1}{2}(m-2)(m-3) - 3p,$$

so that the minimum number of cusps for a self-dual curve is $m - 2$, and there is hence just one self-dual curve for every value of m corresponding to each value of p from zero up to the value given above.

Moreover, since

$$k = m - 2 + 2p \leqq \tfrac{1}{3}m(m-2),$$

the lowest order of a self-dual curve of given deficiency $p(>0)$ is the smallest integer satisfying the inequality

$$m \geqq \frac{5 + \sqrt{24p+1}}{2}.$$

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