happen to read the volume under review may not base upon it a general condemnation of the utterances of philosophers concerning mathematics, but will give themselves an antidote in the form of such books as Russell's Scientific Method in Philosophy or Holt's Concept of Consciousness.

ARNOLD DRESDEN.

Homogeneous Linear Substitutions. By HAROLD HILTON, M.A., D.Sc. Oxford at the Clarendon Press. 1914. Pp. 184.

PROFESSOR Hilton's book is a welcome addition to the text-book literature on the subject of linear substitutions. In the preface the author states that he has "attempted to put together for the benefit of the mathematical student those properties of the homogeneous linear substitution with real or complex coefficients of which frequent use is made in the theory of groups and in the theory of bilinear forms and invariant factors."

The first four chapters, comprising a little more than half of the book, are intended to form an introduction to the whole subject. In the first chapter, which is much the longest in the book, the ordinary method of transforming the general substitution into the normal and canonical forms by means of the poles is shown and the simpler properties of symmetric, orthogonal, unitary, and Hermitian substitutions are given. In the second the author gives a very brief account of invariant factors* and develops the second canonical form which is the direct product of substitutions of the type

$$x_1' = x_2, \quad x_2' = x_3, \cdots, x_{r-1}' = x_r,$$

$$x_r' = e_1x_1 + e_2x_2 + \cdots + e_rx_r.$$

In the third chapter devoted to bilinear forms the Hermitian forms play a prominent part.

To the student who comes to the subject for the first time the fourth chapter on Applications will be one of the most interesting in the book. Illustrations from the theory of equations, from differential equations, from the theory of maxima and minima, from geometry, and from mechanics serve to show the wide range of application of the subject.

* Following Bromwich, Hilton uses the term "invariant factor" instead of "elementary divisor."

Not the least interesting of these applications is the development of the theorems of Thompson and Bertrand on the kinetic energy of systems acted upon by impulses.

The last five chapters are somewhat more advanced in character and are concerned principally with the more detailed properties of symmetric, orthogonal, and Hermitian substitutions. Two brief chapters are devoted to permutable substitutions and to families of bilinear forms.

The book contains a large list of examples which, on the whole, serve to illustrate rather than to extend the subject. For the greater part of the examples indications of the methods of solution are given.

The book contains so much material that has not heretofore been available in one place that it may seem out of place to criticize it for what it does not contain. Nevertheless one can not help feeling that the author has taken his title too literally. For example, it contains no hint of the relation of homogeneous to non-homogeneous substitutions, though in many fields, as in projective geometry and theory of functions, the non-homogeneous form is frequently more convenient.

Again, the reader who is familiar with the group theory will wonder why the word "group" has been excluded from the body of the book even though, as the author says, this phase of the subject has been well treated by Burnside. The introduction of even the elements of the group theory would have given to the book a symmetry and a completeness that it does not now have. Moreover, even Burnside's excellent treatise omits much important material that is to be found in the best continental texts, Weber's Lehrbuch der Algebra, for example. This is particularly true of the fundamental theorems relating to invariants of groups of substitutions of which Hilbert's theorem on the finiteness of form systems is the completest generalization. This gap for English-speaking students has recently been filled in part by the publication of Miller, Blichfeldt, and Dickson's Finite Groups.

The treatment of permutable substitutions, covering as it does a scant seven pages, of which nearly two pages are taken up by examples, could well have been amplified. Of the equations of Weierstrass and Dedekind which play an important part in the theory of algebraic integers as well as in the theory of complex number systems no mention is made.

Likewise the beautiful theorem which asserts that two permutable substitutions must have a common pole is omitted.

The author follows Jordan, Klein, and Burnside in writing the accented, i. e., the new, variables on the left, and writes the factors of a product in such order that multiplication of substitutions is effected through columns-by-rows multiplication of the matrices.

In spite of the omission of some things that the reviewer would have liked to see included, the book is a noteworthy contribution to a subject that is of increasing importance to students of mathematics. It is well written, though the condensed notation makes it rather difficult reading for the beginner in the subject.

ERNEST B. SKINNER.

MATHEMATICS AT AN ITALIAN TECHNICAL SCHOOL.

THE regular courses in the Italian technical schools are five years long, most of the work being prescribed. The schedule is not exactly the same in all of them, yet the general scheme followed can be understood by following the plan of instruction in any one of them. For this purpose we choose that at Milan (Reale Istituto Tecnico Superiore di Milano), which includes a two-year course in mathematics, physics, and chemistry and also in Italian and two foreign languages as preparatory to the regular three-year courses in mechanical, electric, civil, and mining engineering, architecture, industrial, physical, and electric chemistry, and a four-year normal course. Candidates for admission must have a diploma from a recognized Italian secondary school or equivalent credentials. In mathematics they must be familiar with plane and solid geometry, plane trigonometry, algebra including determinants, theory of equations and graphical processes, and the elements of projective geometry.

The following outline applies to all the students except those in architecture, who have briefer courses in all the subjects mentioned.

During both terms of the first year there are three weekly lectures and one hour of exercises in plane and solid analytic