

The standard of typography is high. A few misprints may be noted however: On page 3, line 7,  $\alpha n$  should read  $\alpha_n$ ; on page 30, line 6, for  $y \pm 2kn$  read  $y \pm 2k\pi$ ; on page 59, last line, read  $a_k$  for  $a_*$ ; on page 85 in line 19,  $a_1d - a_1d$  should read  $ad_1 - a_1d$ . The proof of page 106 has not been well read. There occur three notations for the same function on this page, viz.,  $g(x)$ ,  $f'(x)$ , and  $f_2(x)$ . The order of  $f$  is  $m$  at the top of the page, is changed to  $n$  at the middle and so used in two determinants, there being no comment on the change, and the order  $m$  is restored at the bottom of the page. In line 3 from the bottom  $(1/n^{n-1})R(f_2, f_1)$  should be  $(1/n^{n-2})R(f_2, f_1)$ . Moreover it is not good usage, we believe, to begin a sentence with a mathematical symbol instead of a capitalized word, as is done in the theorem given at the top of this page. In line 11 of page 115 the last  $\beta$  in the line is wrong font. On page 120 in line 15,  $2\sqrt{(a_1^2 - a_0a_1)/a_0^2}$  should be  $2\sqrt{(a_1^2 - a_0a_2)/a_0^2}$ . In line 3 from the bottom of page 122 read  $a_4$  for  $a_6$ . In line 2 of page 123 read  $x_3x_4$  for  $x_3x_6$ . The numbering of the formulas in the region of page 123 is confused. Equation (27) referred to in line 6 of this page does not occur in the chapter. This renders line 16 on page 125 unintelligible although it may be a misprint of "Nun liefert (21) wegen (23)." On page 213 in line 4 read  $y_2$  for  $y_2'$ .

In the way of general criticism the reviewer thinks it might be urged that the treatment of invariants in the book is much too brief. Quite probably this subject is to be expounded at greater length in the volumes on geometry. But if it could have been found feasible to introduce the notions of invariancy in connection with the solutions of the equations of orders less than 5, at sufficient length to show, for instance, that the roots of the resolvent cubic of the quartic equation are irrational invariants of index 2, the rôle of invariants in the elements of algebra would have been rendered more evident.

O. E. GLENN.

*Solid Geometry.* By WILLIAM BETZ, A.M., and HARRISON E. WEBB, A.B. With the editorial cooperation of PERCY F. SMITH. Ginn and Company, 1916. xxii+177 pp. Price \$0.75.

ON account of the existence of so many other interesting and important topics in mathematics which can be offered to the

man entering college there is a tendency to drop solid geometry from the curriculum altogether or to relegate it to the high schools. As the primary reason for the study of this subject we may assign the training in space perception and the additional knowledge of the universe which comes to the student; as a secondary, the additional training in logical thinking and expression. Since the former can be realized in a comparatively few lessons and the study of plane geometry *should* suffice for the latter, it is necessary that the colleges which offer this course to Freshmen be able to give an account of the faith which is in them.

It is true that the study of plane geometry is too often a mere memorizing of certain stock propositions rather than a training of the logical faculty, and that many freshmen show a lamentable ignorance of all methods of reasoning including those supposed to be geometrical; but the unbeliever will ask the pertinent question as to whether a really scientific course can be appreciated by the man entering college or whether it is better to postpone further study of deductive geometry until the junior or senior year. Certainly the great majority of texts which flood the market are ill-adapted for a course in proper reasoning. A not over-critical examination of one of the most popular texts in solid geometry shows errors in the statements or proofs of more than one third the theorems. In the case referred to it is probably the result of ignorance; but the authors of a recent text confess that the mass of errors introduced into their treatise is the result of deliberate catering to the infant mind. Unless considerable moral self-restraint is exercised in teaching, the use of such texts with a college class is apt to prejudice the student against mathematics if not against the instructor.

It ought to be possible in America, as it is in some other countries, to put geometry, both plane and solid, into interesting and understandable form without sacrificing logic. For solid geometry, Betz and Webb have done this at least as successfully as in any book in English that has come to the reviewer's attention. It is one of three or four texts which seem possible as a basis for a college course and it seems admirably adapted for high school use. Among the excellent features of the book are: a brief preliminary intuitional introduction to three-dimensional thinking; the grouping together of the fundamental axioms and some of the more intuitionally

obvious propositions as preliminary statements on which to base later demonstrational work; as adequate a treatment of the incommensurable as one could expect in an elementary course; sketches of proofs (such as that in regard to the volume of a cylinder) which might be made rigorous were the proper tools and the time available; a brief introduction to coordinates in space; a careful selection of propositions and exercises, an interesting style, and attractive typography.

While the great majority of the errors that are the bane of the ordinary text in solid geometry are here absent, some have been carried over to furnish targets for the critical mathematician. It may be well to cite here one each of three or four types. (1) In No. 592 concerning polyhedrons it is stated that "the lines of intersection of the bounding planes are called the edges; the points of intersection of the edges, the vertices." However, there may be many lines of intersection of the planes which are not edges and many points of intersection of the edges which are not vertices. (2) In proving (No. 555) that, if one of two parallel lines is perpendicular to a plane, the other is also, it is necessary first to prove that the second line meets the plane. (3) The notion of half-plane must be introduced into the discussion of dihedral angles. Planes *will* extend beyond a line (see No. 559) whether we wish them to or not. The treatment of No. 579 needs an entire revision; among other criticisms it may be noted that the distance from a point to a half-plane face of a dihedral angle has not been defined and can not be defined in any usable manner. In place of this theorem it would be better to introduce V of page 372 and the notions connected therewith. (4) In proving the theorem in regard to the volume of a triangular prism (No. 695) the bases of the two prisms are made to coincide. The question as to whether the prisms will then be on opposite sides of the coinciding bases or on the same side is one of order. To avoid this dilemma a mid-section parallel to the bases might be introduced. While no adequate treatment of the notion "order" is possible in an elementary text, its discussion in such a problem as this and of the orders of the face and dihedral angles of two vertical polyhedral angles (No. 811) warrants a much more careful and extended treatment. And are not the words "same order" in No. 870 used in an entirely different sense from that implied in No. 810?

Professor Smith has rendered a distinct service to the

mathematics of the country by his editing of several texts; this new volume should share in the wide recognition of worth accorded the series.

R. G. D. RICHARDSON.

*The Calculus.* By E. W. DAVIS and W. C. BRENKE. Edited by E. R. HEDRICK. New York, The Macmillan Company, 1913. xx+383+63 pp.

“THIS book attempts to preserve the essential features of the calculus, to give the student a thorough training in mathematical reasoning, to create in him a sure mathematical imagination, and to meet fairly the reasonable demand for enlivening and enriching the subject through applications at the expense of purely formal work that contains no essential principle.”

This is the closing sentence of the preface. It sets forth four things that the authors attempted to do in writing the book. Probably every author of a calculus consciously attempts the first two. An examination of the current texts however reveals but little evidence that the last two have received adequate attention, although there is a clearly defined tendency towards a fuller recognition of their importance. While the formal type of calculus is pretty definitely standardized, there is no generally recognized norm for one of the type here under review. Accordingly a book of this kind is more difficult to write, and also more difficult to teach, than one of the former kind.

It is obvious to any one at all familiar with teachers of college mathematics that the genus is made up of two clearly defined species; namely, those who reverence the symbol and those whose main interest is in the thing symbolized. This book is obviously and confessedly not for the former. It makes its appeal to those who want our students of calculus to realize that the subject is not primarily a formal one, but that it is vitally connected with physical phenomena and represents an important and significant intellectual achievement of the race. For example, instead of devoting a large amount of space to a discussion of the artifices for integration, the authors have presented integration as a process of reversal of rates. They have done this admirably and have brought home to the student with clearness and force what the process is and why it is important for him to study it. And that is the