

duction to the general theory of Cremona transformations is inserted, but so brief and containing so many statements without proof that its value is much less than could justly be expected. A fairly full discussion of the (2, 2) case is given, also an outline of the (2, 3) and (2, 4) cases, but no claim is made for completeness. A brief statement concerning Segre's work on the resolution of singularities by means of quadratic transformations is misleading. The problem was by no means completely solved by Segre.

With the tremendous amount of new material that has been contributed to this field during the last few decades it is difficult to develop a systematic theory without expanding into several volumes. The editor has endeavored to present these new phases in as near the same degree of completeness as was adopted by the author toward the corresponding field when the last preceding edition was written. This has, on the whole, been accomplished. The same style has been followed and a book has been produced, which "forms, it is hoped, a concise and comprehensive survey of tri-dimensional euclidean geometry, both algebraic and differential."

VIRGIL SNYDER.

*Hermann Grassmanns gesammelte mathematische und physikalische Werke.* Herausgegeben von FRIEDRICH ENGEL. Bd. 3: Teil 1, *Theorie der Ebbe und Flut und Abhandlungen zur mathematischen Physik*, 353 pp., 1911 (herausgegeben von JUSTUS GRASSMANN und FRIEDRICH ENGEL); Teil 2, *Grassmanns Leben*, xiii + 400 pp., 1911 (geschildert von FRIEDRICH ENGEL). Leipzig, B. G. Teubner.

GRASSMANN'S works on mathematics and physics are finished after some twenty years of editorial labor.

Although Grassmann's style was such as to repel readers from the two *Ausdehnungslehren*, those works have slowly penetrated into the mathematical consciousness of at least a few persons, but the first part of Volume III of his works contains material that is now printed for the first time and, though written fifty or seventy-five years ago, has only now an opportunity to be valuable to the world at large. The world of science has meantime moved far on, and about the only interest in this early work must be historical, not for the history of science, merely for the history of Grassmann.

The most striking thing about the memoir *Ebbe und Flut*,

which was finished by April, 1840, is the great development which Grassmann had already given to his system of geometric analysis. Here we find not only vector addition and subtraction, the scalar and vector products, differentiation by a scalar, and an incisive treatment of mechanical theory by them, but we find also divergence, the linear vector function, elliptic harmonic motion and the integration of vector differential equations, and rotary operators. The work further shows that at thirty years of age Grassmann, with practically no help, had made himself master of Lagrange and, to a considerable extent at least, of Laplace, so that he could improve on both.

The mathematical physics in this volume is from the Nachlass, and is chiefly analytical optics, some of it more like Gibbs's treatment in his lectures than anything we have seen in print.

For composing the life of Grassmann, Engel had at his disposal a number of documents written by Grassmann himself or by members of his family. It was therefore possible to begin the sketch away back in 1634 (!) and to offer a detailed treatment of the early years of this very busy genius. It is interesting to see how systematically he went to work at the university to broaden his knowledge and render it fundamental in a number of subjects. We may therefore be able a little to understand how Grassmann, who started as a theologian, could have been inventing his analysis, writing on physics, composing class texts for the study of German, Latin, and mathematics, editing a political paper, a missionary paper, investigating phonetic laws, writing a dictionary to the Rig-Veda, publishing a translation of the Rig-Veda in verse, and harmonizing folk songs in three voices, at various times of his life and frequently at the same time—in addition to carrying on successfully his regular work as a teacher and administrator and bringing up nine of his eleven children.

Engel's life of Grassmann is written in a sound critical spirit, there is neither laudation nor condemnation of its subject, merely a connected and sympathetic history of him, from which the reader may get instruction and interest and inspiration. Few biographers in science have had a harder task, for few scientists have had a wider range of activity than Grassmann. We should all admit our deep obligations to Engel as editor and biographer.

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