

When $n = 2$ and the elements of the table are real the series of reduced tables may be arranged, one for each interval, as a single parameter λ ranges both ways from a given value λ_1 defining a series of intervals such that

$$0 < \cdots < \lambda_{-2} < \lambda_{-1} < \lambda_1 < \lambda_2 < \cdots < \infty, \\ (\lim \lambda_i = \infty, \lim \lambda_{-i} = 0).$$

For the general case the reduction is accomplished by means of the fundamental inequalities of Minkowski. The two fundamental problems of finding the units of a realm and of separating the ideals of a realm into classes of equivalent ideals are made to depend upon the reduction of a base.

Three notes, the first on the application of the theory of moduli to periods of functions, the second a study of the realm $K(\sqrt{82})$, and the third a brief account of congruences with respect to an ideal and with respect to the norm of an ideal, occupy the last twenty-four pages of the book.

M. Châtelet modestly disclaims for the book any originality so far as material is concerned. But if it contains no hitherto unpublished results, the treatment is sufficiently novel to make the book a noteworthy contribution to the literature of algebraic numbers. The bringing together of the algebraic analysis of Hermite and the geometrical researches of Minkowski as aids to the development of the brilliant conceptions of Kummer and Dedekind is an achievement for which the mathematical world owes much to the author.

The book as a whole is well written, though at times it is brief almost to the point of obscurity. For the ordinary reader its value would have been greatly enhanced by additional concrete illustrations, and by even a few figures similar to those which illuminate Minkowski's *Diophantische Approximationen*.

E. B. SKINNER.

A Treatise on the Analytic Geometry of Three Dimensions. By GEORGE SALMON. Fifth edition, volume 2, edited by R. A. P. ROGERS. London, Longmans, Green and Company, 1915. xvi+334 pp.

THE second volume of the fifth edition of the *Treatise* begins with families of surfaces, which was Chapter XIII of the fourth edition. The numbering of the chapters has been retained.

The first section has not been changed; the second, on line geometry, has been enlarged from six to forty pages, and made into a new chapter, XIII*a*. It first considers the linear complex, derives the canonical equation, and gives a few important examples. After defining a quadratic complex, the treatment concerns differential properties, including a discussion of singular complexes, singular surface, etc., and then applies the results to the quadratic complex. The singular surface is obtained and a number of properties derived, including the existence of the six special congruences. Then follow a few general properties of congruences of lines, focal surfaces, limiting surface, developables, normal and isotropic congruences. Too many interesting properties are crowded into fine print and given as examples. The part on ruled surfaces is hardly changed.

Section III of Chapter XIII of the fourth edition is also made into a new chapter, XIII*b*. The change consists of an addition of fourteen pages on Lamé's curvilinear coordinates, and Darboux's theorem concerning the intersections of orthogonal surfaces; normal congruences of curves and cyclic systems close the chapter.

In Chapter XIV some bibliographical references are inserted at the beginning. Only one short article on inversion is added. The chapter on the cubic surface is amplified by various minor additions and numerous examples, but the general treatment is not changed.

A number of additions to the theory of the quartic surface have been made; much modern literature has been cited, but by no means all of the important memoirs have been given.

The text of the former edition has been so faithfully reproduced that a serious error concerning plane sections of a ruled surface, passing through a straight line director, is found in the new book (page 203). Two pages are given to Steiner's quartic, one to mapping a quartic surface with a double line on a plane. A short explanation of the complex surface of Plücker, and a fuller discussion of the cones of Kummer connected with the general cyclide, a few pages on the Weddle surface giving an incomplete summary of the later development of this theory, an amplified description of the symmetroid, and the explanation of the Kummer surface rewritten constitute the changes in this chapter.

In the section on the transformations of surfaces, an intro-

duction to the general theory of Cremona transformations is inserted, but so brief and containing so many statements without proof that its value is much less than could justly be expected. A fairly full discussion of the (2, 2) case is given, also an outline of the (2, 3) and (2, 4) cases, but no claim is made for completeness. A brief statement concerning Segre's work on the resolution of singularities by means of quadratic transformations is misleading. The problem was by no means completely solved by Segre.

With the tremendous amount of new material that has been contributed to this field during the last few decades it is difficult to develop a systematic theory without expanding into several volumes. The editor has endeavored to present these new phases in as near the same degree of completeness as was adopted by the author toward the corresponding field when the last preceding edition was written. This has, on the whole, been accomplished. The same style has been followed and a book has been produced, which "forms, it is hoped, a concise and comprehensive survey of tri-dimensional euclidean geometry, both algebraic and differential."

VIRGIL SNYDER.

Hermann Grassmanns gesammelte mathematische und physikalische Werke. Herausgegeben von FRIEDRICH ENGEL. Bd. 3: Teil 1, *Theorie der Ebbe und Flut und Abhandlungen zur mathematischen Physik*, 353 pp., 1911 (herausgegeben von JUSTUS GRASSMANN und FRIEDRICH ENGEL); Teil 2, *Grassmanns Leben*, xiii + 400 pp., 1911 (geschildert von FRIEDRICH ENGEL). Leipzig, B. G. Teubner.

GRASSMANN'S works on mathematics and physics are finished after some twenty years of editorial labor.

Although Grassmann's style was such as to repel readers from the two *Ausdehnungslehren*, those works have slowly penetrated into the mathematical consciousness of at least a few persons, but the first part of Volume III of his works contains material that is now printed for the first time and, though written fifty or seventy-five years ago, has only now an opportunity to be valuable to the world at large. The world of science has meantime moved far on, and about the only interest in this early work must be historical, not for the history of science, merely for the history of Grassmann.

The most striking thing about the memoir *Ebbe und Flut*,