It is to be hoped that in the next edition of this work M. Lebon may be moved to give a list of papers and books which have been inspired by Henri Poincaré's suggestions and discoveries.

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SHORTER NOTICES.

First-Year Mathematics for Secondary Schools. By ERNST R. BRESLICH. Chicago, The University of Chicago Press. 1915. 344 pp.

ABOUT six hundred fifty years ago Roger Bacon gave voice to his feelings with respect to the teaching of mathematics, and this voice was in no respect uncertain nor was it at all lacking in emphasis. His words may be found in the Opus Majus, in the Opus Tertium, and in the manuscripts as yet unpublished of his De Communia Mathematicæ. In the last-mentioned work Bacon says that students are burdened with unnecessary difficulties to such a degree that they come to despise mathematics, whereas, if properly taught, the subject could be understood without any unreasonable expenditure of time; and that the first course in mathematics should not be designated as geometry, arithmetic, and so on, but as the elements of mathematics, a preliminary to the special branches.

What Bacon had to say on this phase of teaching was not new; others had said it before, and thousands have said it since, and after a fashion many have put the idea into practice. And so the effort of Mr. Breslich comes to the teaching profession as merely an ancient one clad in new guise. This does not in the least detract from the laudable nature of the effort, but it serves to give the work a kind of historical setting which assists us in judging of its novelty and its probable effect upon education.

The central idea of the work seems to be to select those features of secondary mathematics which are easily within the reach of beginners, postponing the consideration of the more difficult ones to a later period. As the author puts it, "The simpler principles are best suited for beginners, and may therefore be brought together in an introductory course."

In the pursuit of this idea the author proposes to treat of algebra and geometry at the same time, thus carrying out the ancient idea of fusion to which reference has been made above. He also proposes to consider those "subjects in which practical values are most clearly exhibited," to introduce a certain amount of trigonometry, to avoid "formalism in mode of presentation," and to give the student a "broader mathematical preparation."

With most of this ideal the educational world is generally in sympathy—perhaps with all of it except the mixing of algebra and geometry with no definite system. The following questions, however, will naturally arise in the minds of all who have to consider the book: Has the author carried out the plan successfully? That is, does the book meet the ideals which he has himself laid down? Given the average teacher, will the student, at the end of his work in the high school, be as well grounded in mathematics as he would have been if the work had been arranged on some other plan? Will he appreciate the subject as well or be as apt to continue his study of its higher branches?

In answer to the first question the reader is likely to hesitate before committing himself to an affirmative. He will find fully as much formalism in the early pages (for example, pages 5, 12, 20, 23) as he will find in any of the older types of algebra or geometry; he will find the commutative and associative laws given much earlier than the experience of teachers generally sanctions; he will find, for a work of this nature, an excessive number of definitions; he will find the rules of operation as formally stated as in the more common type of text-book; he will find the euclidean form of greatest common measure, with applications to numbers as large as those in the text-books of two generations ago; he will find such problems as the division of $2,400 into two parts having the ratio of 2 : 1 quite as he would find them in other books; he will find the simple made difficult in various cases, as in such products as $(a - b)c$, $(a + b)(a - b)$, $(a - b)(c + d)$, and $(a - b)^2$, and in the law of signs in multiplication as based on the "turning-tendency" idea; and he will find much the same type of problem that has come down to us from the past, as about a field that is twice as long as wide, and if it were 20 rd. longer and 24 rd. wider the area would be doubled. And when the serious inquirer has finished his reading of the book he will have a feeling of doubt as to whether the plan of improving the first steps in mathematics has been as successful as he had hoped it would be.

And similarly as to the second question. Of course the book can be successfully taught; that is true of any book, provided the right teacher is available. But that a book with what seems to be a forced fusion of essentially different branches of a science, based solely upon the theory of ease of presentation, which theory does not seem to have been carried out—that such a book can be generally successful can hardly be expected.

It seems unfortunate that there should be in the book such statements as that "Pacioli in 1494 was the first to give rules for all processes of addition, subtraction, multiplication, and division" (page 20); that Tartaglia should be spoken of commonly as Fontana (facing page 158), when he himself preferred the former name, as the titles of his books prove; that Vieta or Viète should appear as Vièta (page 210); that the title of Fibonacci's work should be given as *Algebra et almuchabala*, when the manuscript actually begins "Incipit liber Abaci a leonardo filio Bonacij Pisano"; that the student should meet with the name Alkarismi (facing page 213) and with the inexcusable transliteration (unless with diacritical marks) of Al Hovarezmi a few pages later (page 255); that he should be told (page 20) that Diophantus lived about 250, and later (page 281) that he lived in the fourth century; and that numerous other slips of this kind should have been made in preparing the text.

The statement that "the coefficient of any factor in a term is the product of all the other factors of the term" will not seem very clear to a student who is told that 2 is the coefficient of x in the term $2x$. The treatment of negative numbers in Chapter XII will probably not seem to most teachers as clear as those to be found in our common algebras. The assertion that $a \times 0 = 0 \times a$ (page 195) will not seem warranted to those who may use the book, since the commutative law has not been referred to with respect to zero. Such problems as Example 31 on page 251, Example 10 on page 257, and numerous others of this type, will not lead teachers to feel that the author has broken away from the poorest type of inherited puzzles. These and criticisms like these will doubtless strike even the casual reader, and the causes for them will be regretted by all who wish success to any venture of this nature.

In the matter of skillful mathematical typography the book leaves more to be desired than is usually the case.

DAVID EUGENE SMITH.

Les Coordonnées intrinsèques, Théorie et Applications. Par L. BRAUDE. (Scientia, série physico-mathématique, no. 34.) Paris, Gauthier-Villars, 1914. 100 pp. Price 2 francs.

IN 1849 and 1850 William Whewell read two memoirs* on the intrinsic equation of a curve and its applications, before the Cambridge Philosophical Society. The opening paragraph of the first memoir is as follows:

"Mathematicians are aware how complex and intractable are generally the expressions for the lengths of curves referred to rectilinear coordinates, and also the determinations of their involutes and evolutes. It appears a natural reflexion to make, that this complexity arises in a considerable degree from the introduction into the investigation of the reference to the rectilinear coordinates (which are *extrinsic* lines); the properties of the curve lines with relation to these straight lines are something entirely extraneous, and additional with respect to the properties of the curves themselves, their involutes and evolutes; and the algebraical representation of the former class of properties may be very intricate and cumbrous, while there may exist some very simple and manageable expression of the properties of the curves when freed from these extraneous appendages. These considerations have led me to consider what would be the result if curves were expressed by means of a relation between two simple and *intrinsic* elements; the length of the curve and the angle through which it bends: and as this mode of expressing a curve much simplifies the solution of several problems, I shall state some of its consequences." He then considers the curve defined by the equation

$$s = f(\varphi), \tag{1}$$

points out that the radius of curvature follows at once from the relation

$$\rho = \frac{ds}{d\varphi} = F(\varphi), \tag{2}$$

* *Transactions of the Cambridge Philosophical Society,* vol. 8, part 5 (1849), pp. 659, 671; vol. 9, part 1 (1850), pp. 150, 156.