SOME BOOKS ON CALCULUS.

Elements of the Differential and Integral Calculus (*Revised Edition*). By W. A. GRANVILLE. Edited by P. F. SMITH. Boston, Ginn and Company, 1911. xv+463 pp.

Elementary Textbook on the Calculus. By V. SNYDER and J. I. HUTCHINSON. New York, American Book Company, 1912. 384 pp.

The Calculus. By E. W. DAVIS assisted by W. C. BRENKE. Edited by E. R. HEDRICK. New York, The Macmillan Company, 1913. xx+383+63 pp.

Esercizi di Analisi Infinitesimale. By G. VIVANTI. Pavia, Mattei, 1913. vii+470 pp.

GRANVILLE'S Calculus is too widely known both in its original and in its revised edition to require any long notice.* A number of changes have been introduced in the revision and all seem to improve the work as a class drill book. In the number of pages the additions and subtractions exactly balance.

In the preface the author states that in the last few years considerable progress had been made in the teaching of the elements of the calculus and in this revised edition the latest and best methods are exhibited. This statement is entirely incomprehensible to us. So far as we have observed the only important improvement in teaching calculus has been to introduce the calculus earlier in the student's course and so to present it in matter as in time that it may be of greater use to the student in his courses on physics and mechanics. Granville's book veers not the slightest toward this point, no more in the revised than in the original edition.

* For a review of the original see E. B. Van Vleck, this BULLETIN, volume 12, pages 181–187. We are personally out of sympathy with that review because we believe that it represents the view-point of the mathematician catering to the one per cent of the students of calculus who will possibly be pure mathematicians rather than the point of view of the teacher of mathematics who sets his heart on doing the maximum good to the maximum number and who regards mathematics through calculus as essentially the handmaiden of the theoretical and applied sciences. We believe that the aristocratic movement has passed its zenith and is giving way to a less selfish and more democratic point of view,—and we daresay the earlier reviewer is in sympathy with the change.

The calculus is taught to such a large number of students in so many institutions that there is no particular reason why any teacher who has a prominent position cannot find or should not find a publisher for his own notes on calculus and thus have a text of his own which suits him better than any other. This is sufficient excuse for the appearance of Snyder and Hutchinson's book. The work is short. It could have been made shorter without harm by abridging the 42 pages given to contact and curvature, singular points, and envelopes. The most natural book with which to compare Snyder and Hutchinson's is Osborne's (revised edition, 1908). The two are a good deal alike; they give the calculus which is really needed and give it in direct teachable form,—which must be balm to the souls of those that are bored by the modernization of calculus toward rigor, or toward "practical mathematics," or toward the so-called "mixed method."

In their preface the authors call attention to the pressure toward shortening the course in mathematics, they cite the appearance of books on calculus for engineers, physicists, chemists, and so on, and state that it is in recognition of this pressure that they have written. It is good that they are alive to the advisability of adapting calculus to the students who take it; we should all be alive to that fundamental principle of educational justice. But is there any real pressure to shorten the course in mathematics? Is not the pressure rather to get the kind of mathematics the student, in the opinion of engineers, etc., needs? And there is plenty of that kind. Is not the shortening merely an indirect result due to the fact that we will not give the student that which others think he needs and that they therefore diminish his time with us so that they may give him what, in their opinion, he needs more than what we would offer him in any additional time allowed to us?

We may quote from the introduction to Perry's Elementary Practical Mathematics: "Academic methods of teaching mathematics succeed with about five per cent of all students, the small minority who are fond of abstract reasoning; they fail altogether with the average student. Mathematical study may be made of great value to the average man if only it is made interesting to him." Here is the real reason for the pressure there is upon us. We deal in the abstract and in the rigorous; the average person does not, and to a certain extent cannot. We teach the wrong way,—let us quote again: "There is always a difficulty in obtaining competent teachers (of practical mathematics). Any man who has learnt pure mathematics is thought by himself and others to be fit to teach, whereas his very fondness for and his fitness to study pure mathematics make it difficult for him to understand the simple principles underlying the new method. The average boy cannot take to abstract reasoning, and he is called stupid; I think him much wiser than the boy who is usually called clever."

We may not believe any of this stuff, we may force it out of our consideration; but there are many who believe it all, and they will constantly bring it back to our attention. And we cannot compromise more than temporarily by abridging our course; the very abridgment will produce less efficiency in the sort of thing we do teach. Even an average class will take great delight in hard differentiations and integrations, they will rejoice in conquering the difficulty, as I many times observed in the classes of A. W. Phillips at Yale,—provided the class is drilled in differentiation and integration until the majority acquire sufficient technique to make the game interesting. It is ability to do that maintains the interest. When we abridge our course without otherwise changing it we diminish the chances that the student shall become able to do what we teach him. That is the weakness of mere abridging. Diminishing the requirement in Greek for entrance to college killed preparatory Greek as quickly as anything could.

For ourselves, we do not believe in going the whole way with Perry; we believe that some abstract reasoning is good, and with our students prepared as they are when they come to us from the secondary schools a certain amount of abstract reasoning is not only good but possible. If we can follow a short course in calculus from a book like Snyder and Hutchinson's by a considerable course in concrete and practical problems, that may be our best procedure. But if we are to be allowed altogether only a short course, we should make that much less mathematical in the canonical sense; and by doing so we may perhaps be entrusted with a greater allotment of time.

Davis's Calculus is a frank attempt to introduce variety and interest into the calculus. The work therefore has attractive elements; one may easily exclaim: How inspiring to teacher and pupil to have all this constant contact with nature! That the book has bad qualities is obvious to anybody examining it carefully, but it is only after the sad disillusionment of teaching it a year that one can really find out how largely the bad outweighs the good. The book will therefore have many enthusiastic adopters and many speedy rejectors.

The main difficulty is that careless workmanship (or playmanship) permeates the whole in such an insidious fashion that it is partly hidden to the prospective user and always a burden to the actual user. Whether author, assistant author, or editor is responsible for this defect we cannot say; but it is improbable that any real hard cooperation by all upon the whole could have left so many lesions, and we may guess that one brewed the text, another peppered in the exercises, and a third sprinkled in the sage advice to Dear Reader and the gratuitous reflections. The answer book is full of errors, and thus is a great annoyance to the serious student, a corrupter of the careless worker. A table is valueless except as it is accurate, yet inaccuracies are found in the formula for center of pressure (not given in the text) and in the polar equation of the cissoid.

If Davis-Brenke-Hedrick had written a sufficiently original text we could pardon a number of errors, even under triple responsibility; but there is no particular originality about the work. They treat the algebraic function first, both as regards integration and differentiation, and when they come to transcendental functions they carry on the differential and integral calculus simultaneously. But so did Mercer in 1910; and if we may trust a comparative judgment of two books one of which we have not taught, we should unhesitatingly say that Mercer, though bearing but one workman's name, is incomparably the more careful and valuable production. And, to mention no others, Byerly in his Differential Calculus as long ago as 1879 introduced the integral calculus early and carried it along with the differential. It can hardly be expected that Byerly's book as it stands after 35 years should appeal strongly to teachers of the present day; yet its plan has many of the good features of recent books which try to freshen up the calculus.

The authors include a considerable treatment of differential equations as did Mercer in 1910 and Woods and Bailey in their Course in Mathematics, volume 2, some years before. This is not a bad thing to do, for the integration of the simpler differential equations is a natural part of the integral calculus; but I am very strongly of the opinion that a systematic chapter on rectilinear dynamics, such as is found in Byerly's and Osgood's books, is of much more value to the student and teaches him just as much about differential equations. One of the earliest differential equations we meet is the so-called compound interest law, for which our authors use the term snowball law as more " suggestive "; it is too bad that they regard it as more suggestive, for it is very wide of the mark. (The snowball law makes an interesting exercise for engineers; it leads under reasonable assumptions, such as an engineer would make, to an algebraic integral.)

One thing above all others students of mathematics should learn, namely, that it is the business of mathematics to teach them to think and talk coherently. They should have clean-cut definitions and straightforward proofs. If a book contains these, the teacher may safely be trusted to furnish the discussion and to point out the beauties of the landscape; whereas it is difficult for the teacher to furnish the definition and proof with any effectiveness to a class whose text is mostly discussion. The authors' book is long despite its moderate number of pages; for it is printed to a large extent in eight-point type. It could be abridged without substantive omissions.

It is the business of sets of exercises to offer definite and somewhat graded problems to the student; but the exercises in this text are wholly ungraded and in many cases very indefinite in statement. Sometimes there is scarcely a single exercise in a whole set which I should wish to assign to a class, and in every case it is necessary to take the greatest care in the selection of exercises. We recommend a careful reading of the lists of exercises to every prospective user.

The practice of leaving important items for exercises, a plan of which I heartily approve in advanced texts where exercises are hard to find or where we are trying to teach the student research, seems very doubtful for any elementary text. Yet here the area of a surface of revolution, component accelerations, hyperbolic functions, average values, centers of gravity, the element of volume in polar or spherical coordinates are thus left as exercises. It must be a remarkable class that can work out for itself the polar element of volume; it is more than most of us can do to draw a decent figure and give the proof for the class.

We have heard a great deal off and on about the necessity of giving the student power and the spirit of investigation; but this is merely a visionary's ideal, as anybody can see by pondering upon the question: How many of our doctors of philosophy in mathematics in this country or abroad are engaged in real research? If six to eight years of training lapse into desuetude in the case of professional mathematicians, what can you expect to accomplish with sophomores? Put into the text what you want them to know in such form that they can learn it, say I, and then see that they do learn it. And I have heard a very eminent investigator recommend the same sort of thing for candidates for the doctorate.

There is no need of going into the details, whether bad or good, of Davis's text. Suffice it to say that if books are not more carefully written, we shall have to refrain from adopting them from very lack of time to examine them in sufficient detail to make it safe to adopt them; but it is too bad to throw the whole reponsibility upon the user instead of upon the author and publisher, where we previously thought it belonged, at least to a very large extent.

Vivanti's book of exercises, a companion to his Lezioni d'analisi infinitesimale, contains 575 well-selected solved exercises in calculus; there are no applications and no rigorous types. The list should be of value to American writers of texts, but it is difficult to see how it can be of direct use in our classes.

EDWIN BIDWELL WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

NOTES.

THE April number (volume 16, number 2) of the *Transactions of the American Mathematical Society* contains the following papers: "Quartic curves modulo 2," by L. E. DICKSON; "Mixed linear integral equations of the first order," by W. A. HURWITZ; "Prime power groups in which every commutator of prime order is invariant," by W. B. FITE; "On