The Pell Equation. A history of the equation $x^2 - Ay^2 = 1$, table of solutions from $A = 1{,}501$ to $A = 1{,}700$, bibliography, table of continued fractions for $\sqrt{A}$. By EDWARD EVERETT WHITFORD. Lancaster, Pa., The New Era Printing Company, 1912. iv+193 pp.

THIS book presents in a very readable form an historical account of this famous problem. The author first discusses, in considerable detail, various efforts of the ancient Greek and Hindu mathematicians to find rational approximations to square roots, showing how these ultimately depend upon solutions of special cases of the Pell equation.

While these efforts and those of later mathematicians to find square root approximations do form the historical background of this problem, its formulation in general terms was made by Fermat in 1657. The author discusses the contributions of Lord Brouncker and of Euler toward the solution of the general case and justly credits Lagrange with the first rigorous proof of its solvability.

The relation of the Pell equation to the theory of quadratic forms is shown in connection with the more modern methods of Gauss and Dirichlet.

The excellent bibliography, together with the tabulated continued fraction developments of $\sqrt{A}$ from $A = 1{,}501$ to $A = 2{,}012$, as well as the table of fundamental solutions of the Pell equation as indicated in the sub-title, make this a very useful book for the worker in this field.

T. M. PUTNAM.

Die Berührungstransformationen: Geschichte und Invariantentheorie. Zwei Referate, der deutschen Mathematiker-Vereinigung erstattet von H. LIEBMANN und F. ENGEL. (*Jahresbericht der deutschen Mathematiker-Vereinigung.* Der Ergänzungsbände V. Band.) Leipzig, Teubner, 1914. v + 79 pp.

IN the first of these reports (pages 1–14), Liebmann gives an attractive sketch of the historical development of the theory of contact transformations. The second report (pages 15–77) on "Lie's theory of invariants of contact transformations, and its extension" by Engel is an original contribution to the theory in question rather than a report. As Engel points out in his introductory remarks, Lie took great pride in using only such methods as he had originated himself, even where simplifications due to others could be introduced to great advantage. Thus, in the second volume of his great work on transformation groups, he not only took scant notice of Mayer's simple and direct derivation of the theory of contact transformations, but ignored completely the notion of the bilinear covariant of a Pfaff differential expression, due to Lipschitz and applied with great success by Frobenius. As S. Kantor has indicated in several papers published in the *Sitzungsberichte* of the Vienna Academy (1901–03), Pfaff's problem and particularly Mayer's theory of contact transformations are greatly simplified by the use of the bilinear covariant; however, Kantor's papers, besides containing a considerable number of errors, are extremely unsystematic and obscure.

In the present work, Engel gives a very clear and simple exposition, based on the bilinear covariant, of the general theory of contact transformations and their invariants, and the application to partial differential equations, and elaborates the generalization, conceived but never worked out by Lie, of the invariant theory to Pfaff expressions in $2n$ variables. The important work of Engel should prove of great value to investigators in this field.

T. H. GRONWALL.

Veränderliche und Funktion. Von M. PASCH. Leipzig, Teubner, 1914. vi + 186 pp.

THIS little book deals primarily with the fundamental ideas at the basis of the theories of variables and functions and with their applications or illustrations by means of some of the most elementary functions. One finds a treatment of such topics as the following: order relations among numbers and the associated ideas, kinds of mathematical proof as illustrated in the foregoing discussion, point sets, sequences, variables and constants, various rational functions and classes of such functions, continuity and uniform continuity, exponential and logarithmic functions, countable sets, etc. The discussion is interspersed with a considerable number of interesting remarks belonging, one may perhaps say, to the philosophy of mathematics.

R. D. CARMICHAEL.