have also proved that a field of type (9) is self-conjugate* and is such that the flow of energy (Poynting's vector) is along the radius from (ξ, η, ζ, τ) to (x, y, z, t), hence it follows that the different æthereal fields given by (9) form a mutually conjugate system.

There are still many questions to be answered before the present theory can be accepted. If the æther is regarded as built up of elements a law describing the mutual action of two elements is needed. The nature of the macroscopic equations satisfied by the vectors (E, H) in a region occupied by an enormous aggregate of singular curves of æthereal fields has not been ascertained. The field obtained by superposing all the æthereal fields is probably different in character from an ordinary electromagnetic field; it may possibly have some connection with the phenomena of gravitation.

JOHNS HOPKINS UNIVERSITY,
BALTIMORE,
October 27, 1914.

SHORTER NOTICES.

Handbuch des mathematischen Unterrichts. Von W. KILLING und H. HOVESTADT. Band II. Leipzig and Berlin, Teubner, 1913. x + 472 pp.

THE general character of this work was discussed by the reviewer in the BULLETIN, volume 17, No. 5. The vigor which characterized the first volume is unabated in the present one, which is devoted to trigonometry. One third of the volume is devoted to plane trigonometry, an equal amount to solid geometry or stereometry in which plane trigonometry is used freely in dealing with space problems, and the remainder to spherical trigonometry. There is sharp criticism of the ordinary text-books on trigonometry, in which the authors deplore the tendency to limit the field to goniometry and the solution of triangles with and without logarithms. Nor are they satisfied with a criticism of the content, for the methods of proof come under fire as well. As an illustration of the carefulness of their investigation, four proofs of the addition

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* *E*, p. 12; *Phil. Mag.*, Jan., 1914.

theorem for the sine as given in different texts are stated, and thoroughly discussed as to their value, not merely in proving the immediate theorem, but in suggestiveness and applicability to further problems. The author of an ordinary American text-book would not feel flattered if he measured his product by the criteria of this volume. To show the value of trigonometry as a weapon in geometry, various well-known theorems such as the Feuerbach theorem are proved and an entire chapter of 50 pages is given to the application of trigonometry to construction problems. The chapter on teaching the applications of plane trigonometry is rich in suggestions, and could be read with profit by any teacher of the subject.

The section on stereometry covers a wider range than the usual elementary solid geometry. Among the applications is an interesting chapter on the geography of the sky and map making, including mercator and stereographic projection. Some chapters in fine print, such as the calculation of the surface of a curved solid by integration, and the bisection of a tetraedron by a hyperbolic paraboloid, are obviously of more interest to the teacher than to the immature student. The chapter on Euler's theorems giving the relations between the number of edges, faces, vertices, and face triangles of a convex polyedron is valuable, and shows forcibly how closely the authors have scrutinized supposed proofs. In talking about Riemann surfaces they again go beyond the depth of the ordinary student of solid geometry. The deficiency or redundancy of the numerous definitions of a regular polygon seems to be an object of keen attack. The chapter on regular spherical polygons in connection with regular polyedra is excellent and might well be incorporated into elementary texts.

The closing section on spherical trigonometry is no less critical than the earlier parts of the work, although in content it more nearly approximates a text-book on the subject. The proofs of Gauss's formulas and Napier's analogies are carefully considered, and the authors conclude their criticism of existing proofs by a new one of their own. They see no use in developing a mass of formulas which have historical interest only, so special theorems and formulas which play no rôle in the applications are reserved for the last chapter. The deduction of the theorems on the geometry of the sphere from the formulas of spherical trigonometry is a point which teachers might well note. The applications considered are the ordinary ones,—measurement of the earth, astronomical problems, determining one's position at sea, and the like. The last chapter is a discussion of the work of Möbius on spherical triangles whose sides or angles may be greater than 180°, and of the still further extensions of Study. Any reader of this volume will be impressed with the possibilities of a course in trigonometry.

D. D. LEIB.

Théorie des Nombres. Par E. CAHEN. Tome premier: *Le premier Dégré.* Paris, A. Hermann et Fils, 1914. xii + 408 pp.

THE first eight chapters contain an exposition, mainly following the ideas of Helmholtz, of the definition and properties of integers, the four fundamental operations, divisibility, greatest common divisor and least common multiple, and the theory of fractions. Chapter 9 deals with diophantine equations in one and two unknown quantities, and the euclidean algorithm. Chapters 10 to 12 contain an exhaustive treatment of diophantine equations in any number of unknown quantities, and systems of such equations, preceded by an exposition of the corresponding algebraic theories (determinants and systems of linear equations). Chapters 13 to 16 give the theory of linear substitutions and linear and bilinear forms, the number theoretic case, where the coefficients are integers, being always preceded by an exposition of the corresponding algebraic case, where the coefficients are arbitrary. Chapter 17 is concerned with linear congruences, and chapters 18 to 20 contain the fundamentals of the algebra of matrices, followed by the corresponding number theoretic propositions for matrices with integers as elements. The three last chapters give the decomposition of an integer into prime factors, the properties of Euler's function $\varphi(n)$ and some other number theoretic functions, and some remarks on linear congruences with prime modulus.

Misprints are rather numerous, though seldom misleading, and it is difficult to see why the name of Frobenius should be persistently misspelled; one may also well take exception to the narrow and unusual definition of number theory proposed in the introduction. But these are minor criticisms, and the volume should prove of great value as a text-book, since the