

A great many theorems dealing with the limit-behavior of functions were obtained by Du Bois-Reymond in a series of papers dating from 1871 to 1880. These theorems have been collected, recast according to modern requirements of rigor, and amplified by Hardy in No. 12 of the Cambridge Tracts in Mathematics and Mathematical Physics. The work has been done in a manner admitting of no criticism; the treatment is clear and readable; the proofs are accurate and carefully worded.

Questions of the sort considered are generally reducible to a comparison of the rates of increase of positive real functions, as the positive real independent variable becomes infinite (continuously or over integral values); into this form most of the results are thrown by the author. A considerable part of the tract deals with logarithmico-exponential functions,—those obtained by rational operations, the extraction of roots, and the taking of logarithms and exponentials. Such functions have various properties which render their study easy and important; for instance, they always admit of comparison as regards rate of increase, and relations of comparison may under simple conditions be differentiated and integrated.

The tract contains a sketch of applications to such questions as the convergence of series and integrals, asymptotic formulas, the distribution of prime numbers, and the theory of integral functions of a complex variable.

One is impelled to wonder how much of the fairly extensive notation introduced will be found really desirable in actual use of the results. The notions of inferior and superior limit have won a permanent place; Landau's symbols  $O(f)$  and  $o(f)$  have in a short time come into such wide use as apparently to insure their retention. It is doubtful whether any further notation will be found necessary.

The tract is printed with the clearness characteristic of the series. The only typographical inaccuracy noted is on page 42, line 12, where  $e_{\Delta x}$  should be replaced by  $e^{\Delta x}$ .

WALLIE ABRAHAM HURWITZ.

*The Hindu-Arabic Numerals.* By DAVID EUGENE SMITH and LOUIS CHARLES KARPINSKI. Boston, Ginn and Company, 1911. vi + 160 pp.

THE origin and development of our present number system is probably given as little thought as anything that is so

commonly known and used. For persons who are not interested in things mathematical, as well as for those who are thus interested, *The Hindu-Arabic Numerals* furnishes information that is by no means current. The authors have given in concise form the history of the characters that every school boy of modern days learns to use in computation. Beginning with an account of the early ideas about these characters, the writers go on with the details of place value, the symbol zero, and the actual physical forms by which the numbers have been represented at various times. In chapter 5 is begun the account of their introduction into Europe. This westward aggression seems to have been very slow at first because the new numerals did not appeal to the traders and were regarded as a sort of novelty by educators. The complete acceptance of this really wonderful system finally took place in the sixteenth and seventeenth centuries.

J. V. MCKELVEY.

*Vorlesungen über darstellende Geometrie*, Band I. Dr. F. v. DALWIGK. Leipzig, Teubner, 1911. xvi + 360 pp.

DESCRIPTIVE geometry is presented in this book in a thoroughly instructive manner. A noticeable balance is preserved between what might be called theoretical and practical. Methods of treatment characteristic of pure geometry are used freely, but at the same time the technical use of the principles involved is given due attention. The frequent comparison of different methods of making a given construction is worthy of mention. Shadow construction is treated with excellent simplicity and completeness throughout the book. The treatment of the projection of the intersection of two surfaces is such as to give the beginner a good introduction to the theory of space curves.

In the latter part of the text, trihedral angles with applications to astronomy, surfaces of revolution, screw surfaces, stereographic projection, parallel perspective, and rectangular coordinates are taken up in some detail. The last section of the book is an introduction to mechanical drawing.

J. V. MCKELVEY.