Exercices de Géométrie analytique. Par P. AUBERT and G. PAPELIER. Tome Premier. Paris, Libraire Vuibert. 360 pp.

IN this book are given 451 exercises on analytic geometry divided among the following fields: the straight line, 94 exercises; the circle, 69 exercises; parametric equations, 46 exercises; classification of conics, 55 exercises; the general equation of the conic, 31 exercises; centers and diameters of conics, 34 exercises; the general properties of conics, 91 exercises; polar coordinates, 31 exercises. Many of the exercises are accompanied by a figure and about one half of them are solved. In several cases more than one solution is given, so as to illustrate different methods of approaching the same problem. The authors have assumed the student to be familiar with harmonic properties, projective transformations, and similar topics which are given in the United States in a second course in analytics. Homogeneous coordinates are not used, and thus many of the problems are solved by rather tedious methods. In our American colleges the book could be used to good advantage in a second course in analytics in conjunction with such a book as Salmon's Conic Sections. The problems are of too difficult a nature for a first course as given in our institutions.

F. M. MORGAN.

Orders of Infinity. The "Infinitärcalcül" of Paul Du Bois-Reymond. By G. H. HARDY. Cambridge University Press, 1910. viii + 62 pp.

MANY problems of mathematical analysis are adequately treated by consideration of the fact that a function does or does not approach a limit, where we may understand the notion of a limit to include the real infinity, with or without sign, and the complex infinity. Other questions, however, demand more refined investigations,—on the one hand when a limit exists and an estimate is necessary of the rapidity of approach to the limit; on the other hand when no limit exists and some notion of the behavior of the function is still requisite. As early as 1821, Cauchy recognized the usefulness of such considerations in his accurate definition of la plus grande des limites and la plus petite des limites,—today more commonly known as superior and inferior limits.

A great many theorems dealing with the limit-behavior of functions were obtained by Du Bois-Reymond in a series of papers dating from 1871 to 1880. These theorems have been collected, recast according to modern requirements of rigor, and amplified by Hardy in No. 12 of the Cambridge Tracts in Mathematics and Mathematical Physics. The work has been done in a manner admitting of no criticism; the treatment is clear and readable; the proofs are accurate and carefully worded.

Questions of the sort considered are generally reducible to a comparison of the rates of increase of positive real functions, as the positive real independent variable becomes infinite (continuously or over integral values); into this form most of the results are thrown by the author. A considerable part of the tract deals with logarithmico-exponential functions,—those obtained by rational operations, the extraction of roots, and the taking of logarithms and exponentials. Such functions have various properties which render their study easy and important; for instance, they always admit of comparison as regards rate of increase, and relations of comparison may under simple conditions be differentiated and integrated.

The tract contains a sketch of applications to such questions as the convergence of series and integrals, asymptotic formulas, the distribution of prime numbers, and the theory of integral functions of a complex variable.

One is impelled to wonder how much of the fairly extensive notation introduced will be found really desirable in actual use of the results. The notions of inferior and superior limit have won a permanent place; Landau's symbols $O(f)$ and $o(f)$ have in a short time come into such wide use as apparently to insure their retention. It is doubtful whether any further notation will be found necessary.

The tract is printed with the clearness characteristic of the series. The only typographical inaccuracy noted is on page 42, line 12, where $e_{\Delta x}$ should be replaced by $e^{\Delta x}$.

WALLIE ABRAHAM HURWITZ.

The Hindu-Arabic Numerals. By DAVID EUGENE SMITH and LOUIS CHARLES KARPINSKI. Boston, Ginn and Company, 1911. vi + 160 pp.

THE origin and development of our present number system is probably given as little thought as anything that is so